

Journées Scientifiques du GdR MoMaS
Villa Clythia, Fréjus St-Raphael
14-16 novembre 2007

**milieux poreux non saturés:
approche micromécanique**

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remerciements: E. Lemarchand, J. Sanahuja, F. Skoczylas

plan de l'exposé

- Principes généraux de “micro-poromécanique”

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 - régime saturé
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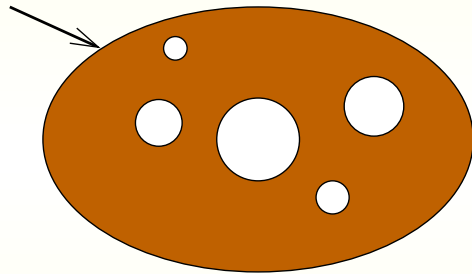
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Elasticité linéaire d'un poreux sec

$$\boldsymbol{\sigma} = \mathbb{C}(\mathbf{z}) : \boldsymbol{\varepsilon}$$

$$\mathbb{C}(\mathbf{z}) = \begin{cases} \mathbb{C}^s(\mathbf{z}) & \text{in } \Omega^s & \text{(solide)} \\ 0 & \text{in } \Omega \setminus \Omega^s & \text{(espace poreux)} \end{cases}$$

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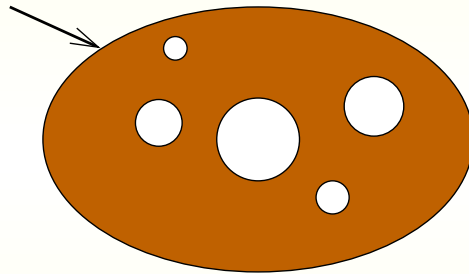


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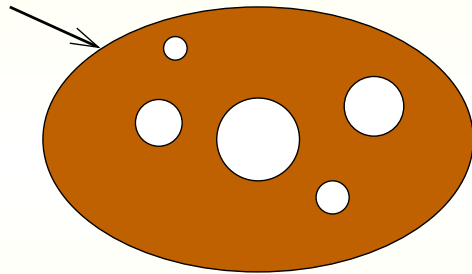
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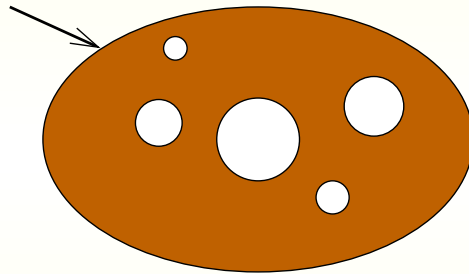
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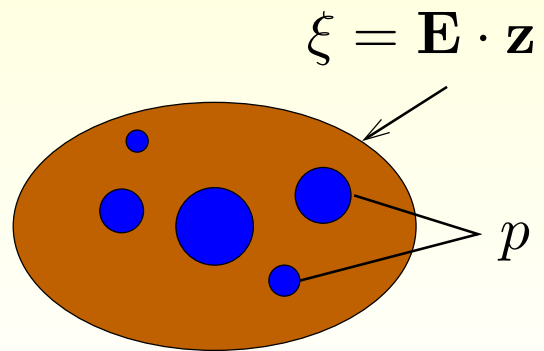


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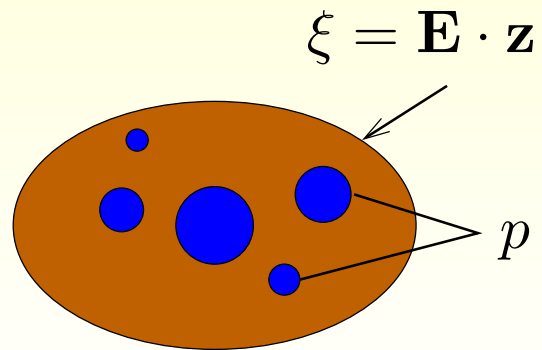


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Poroélasticité linéaire



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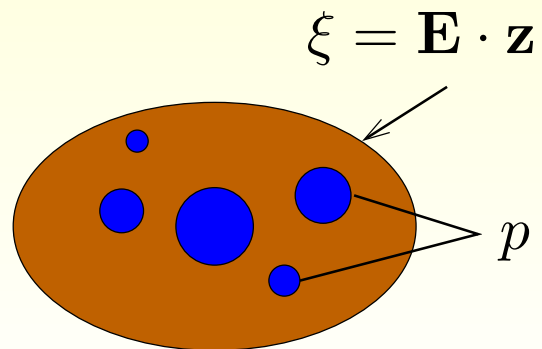


$$\boldsymbol{\sigma} = \mathbb{C}(\mathbf{z}) : \boldsymbol{\varepsilon} + \boldsymbol{\sigma}_o(\mathbf{z})$$

$$\mathbb{C}(\mathbf{z}) = \begin{cases} 0 & \text{in } \Omega^p \\ \mathbb{C}^s & \text{in } \Omega^s \end{cases}$$

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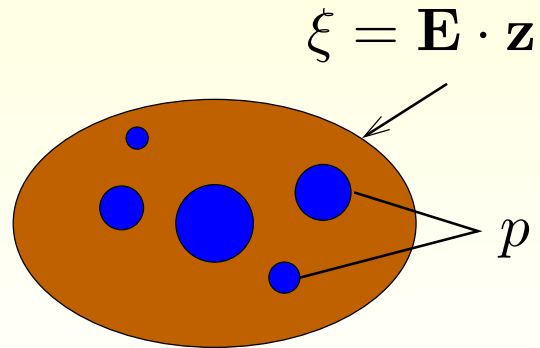
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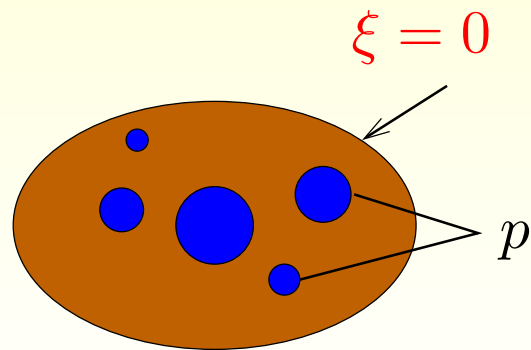
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$$\mathbf{B} = \varphi \mathbf{1} : \overline{\mathbb{A}^p}$$

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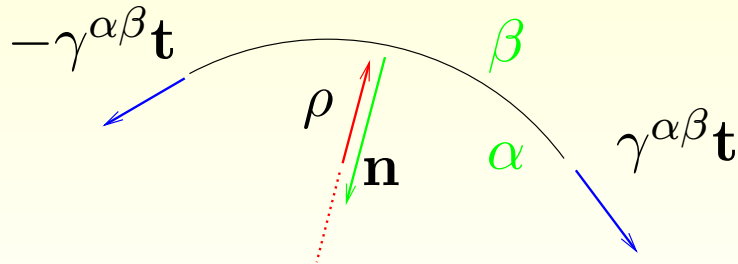


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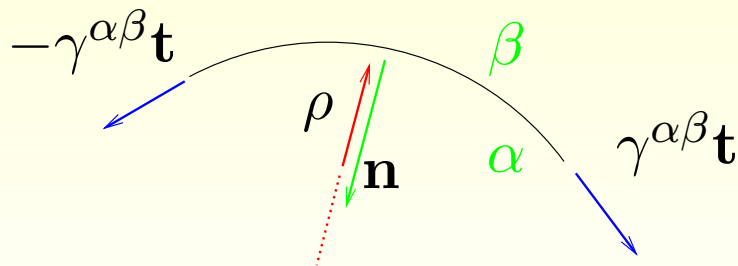
gonflement empêché ($\mathbf{E} = 0$) $\Rightarrow \Sigma = -p\mathbf{B}$

Interaction mécanique entre phases

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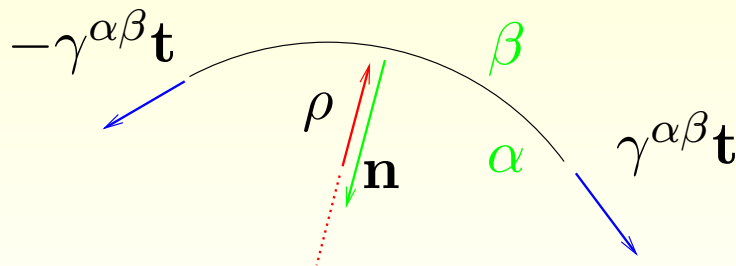
Interaction mécanique entre phases



$$[\boldsymbol{\sigma}] \cdot \mathbf{n} + \frac{\gamma^{\alpha\beta}}{\rho} \mathbf{n} = 0$$

(interface cylindrique)

Interaction mécanique entre phases

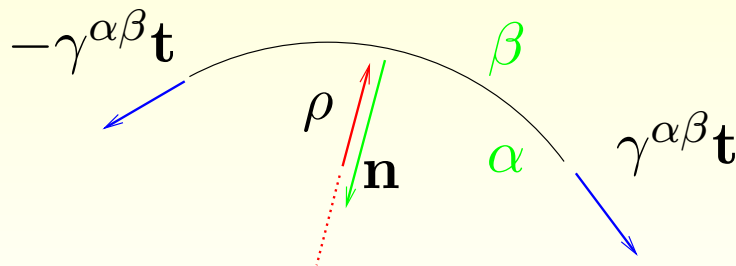


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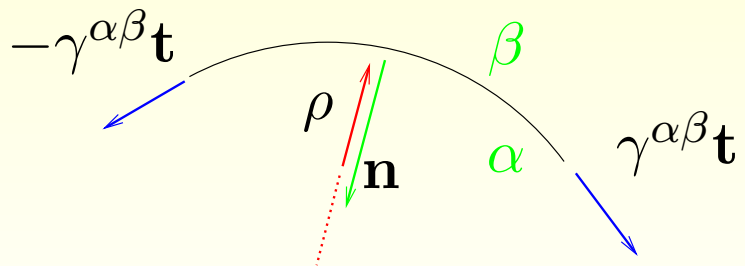
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$$\boldsymbol{\sigma} \cdot \mathbf{n} = -\left(P - \frac{\gamma^{\alpha\beta}}{\rho}\right) \mathbf{n}$$

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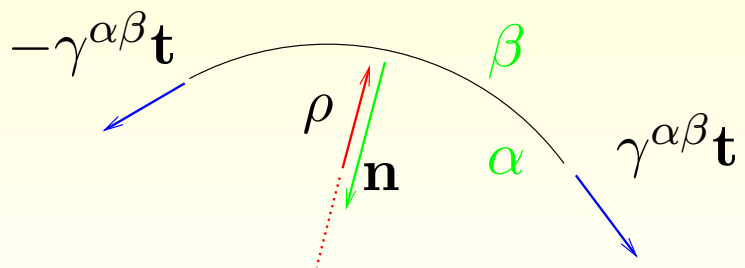
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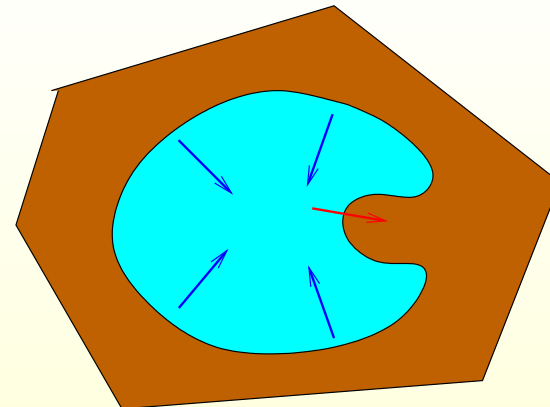
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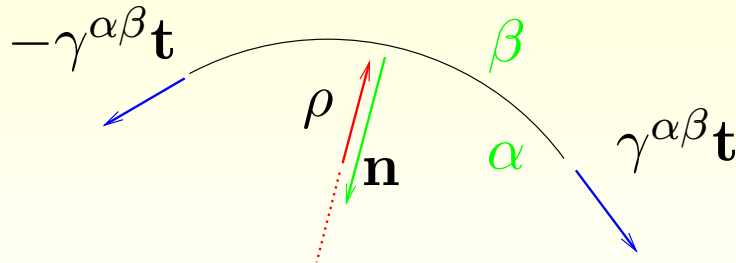
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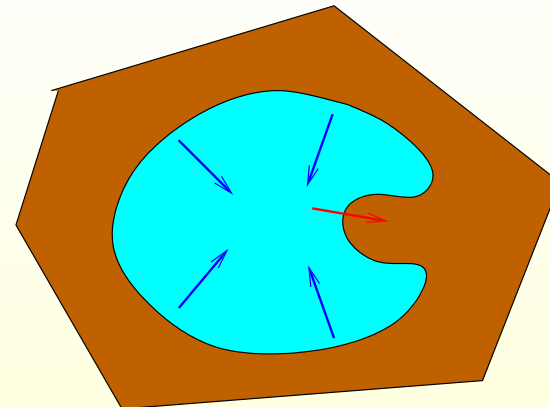
$$[\boldsymbol{\sigma}] \cdot \mathbf{n} + \gamma^{\alpha\beta} (\mathbf{b} : \mathbf{1}_T) \mathbf{n} = 0$$

3D généralisation ($\mathbf{b} = -\text{grad } n$)

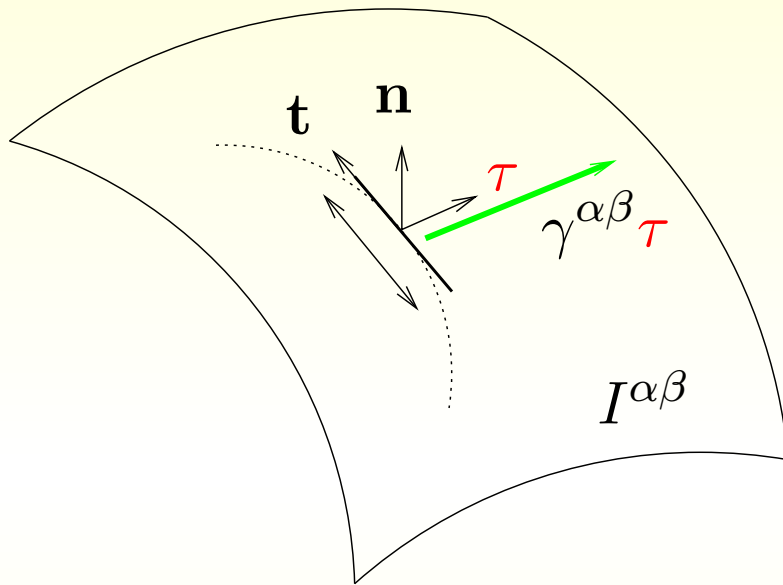
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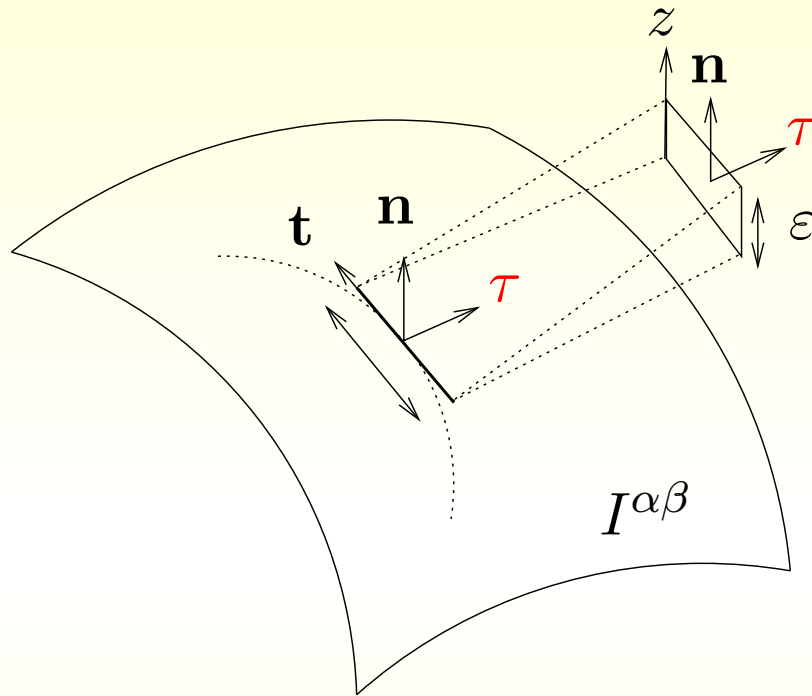
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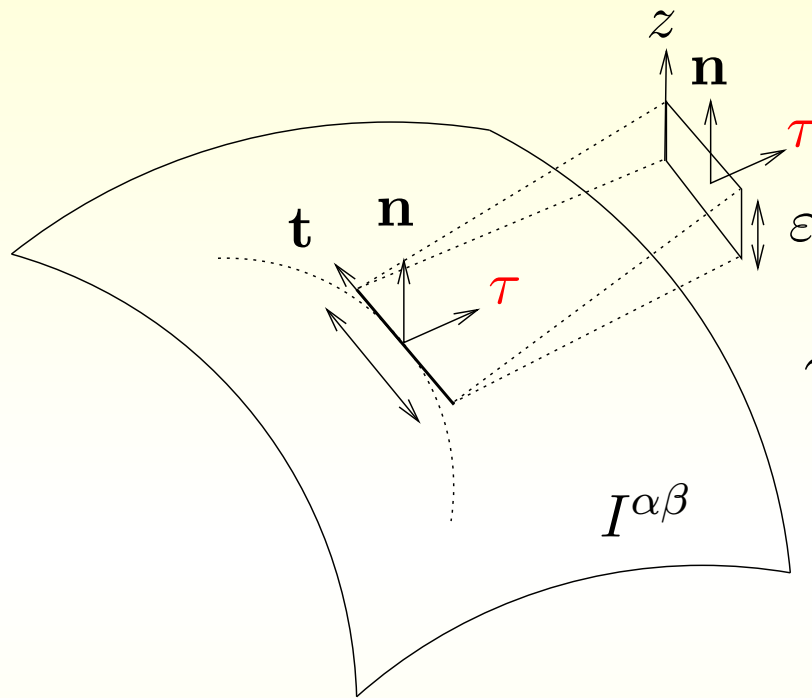
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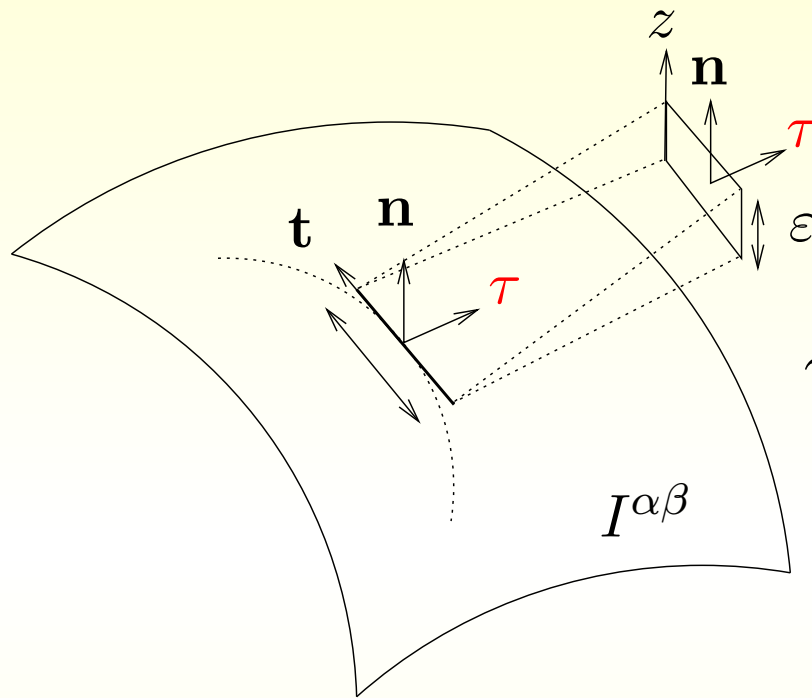


Représentation des efforts intérieurs



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contrainte de membrane \rightarrow contrainte de Cauchy :

$$\boldsymbol{\sigma}^{\alpha\beta} = \gamma^{\alpha\beta} \delta_{I^{\alpha\beta}} \mathbf{1}_T$$

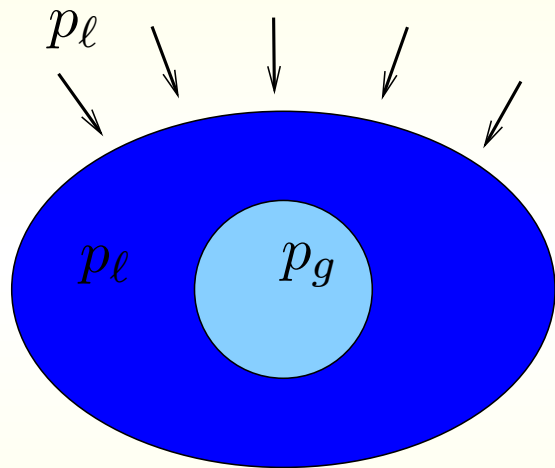
Contrainte macro./contraintes micro.

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- règle de moyenne sur les contraintes : $\Sigma = \bar{\sigma}$?

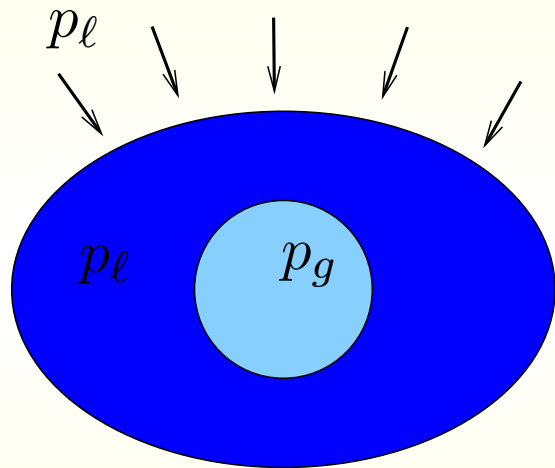
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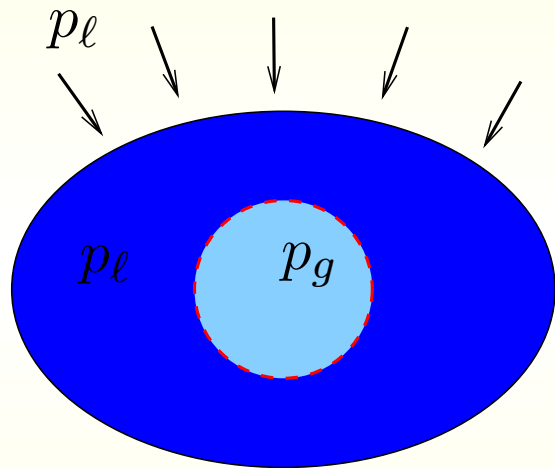
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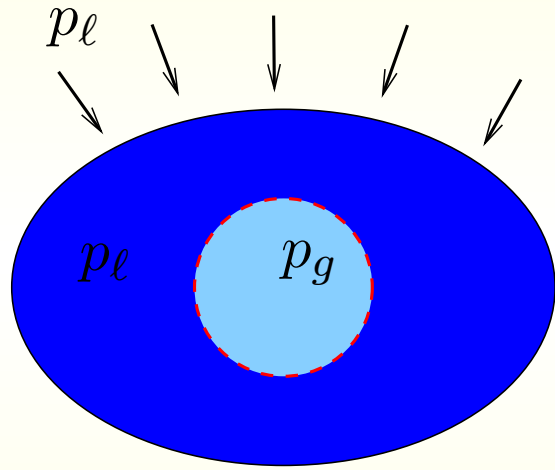
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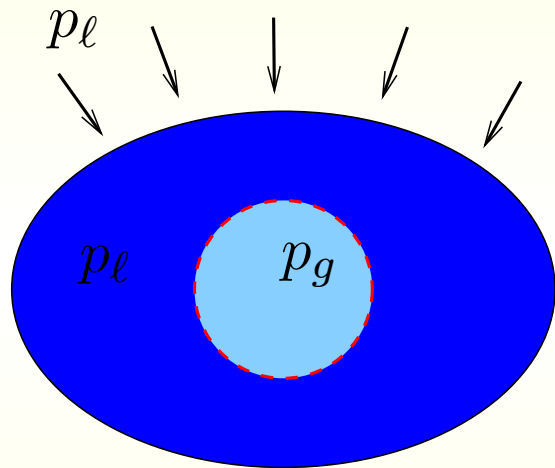


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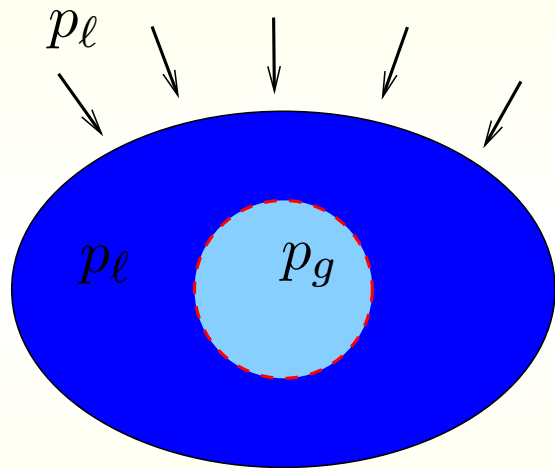


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$$\int_{I^{lg}} \gamma^{lg} \mathbf{1}_T dS_z = \frac{4\pi R^3}{3} \times \frac{2\gamma^{lg}}{R} \mathbf{1}$$

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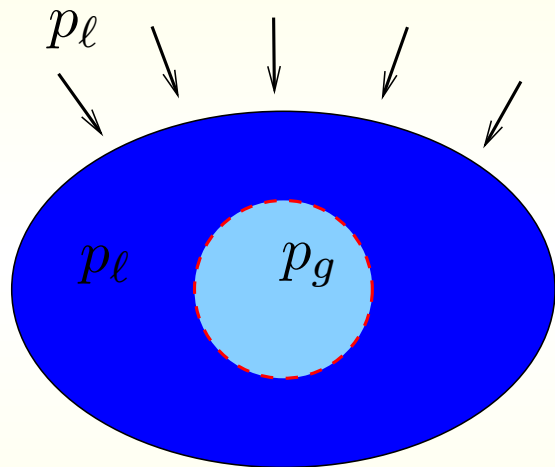
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A propos de l'état initial

$$\boldsymbol{\sigma}(\mathbf{z}) = \mathbb{C}(\mathbf{z}) : \boldsymbol{\varepsilon} + \boldsymbol{\sigma}_o(\mathbf{z}) \Rightarrow \boldsymbol{\Sigma} = \mathbb{C}^{hom} : \mathbf{E} + \overline{\boldsymbol{\sigma}_o} : \mathbb{A}$$

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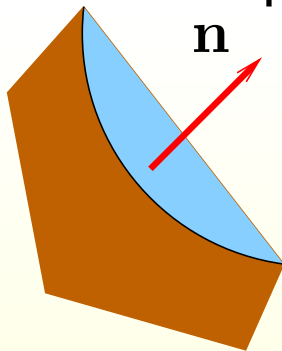
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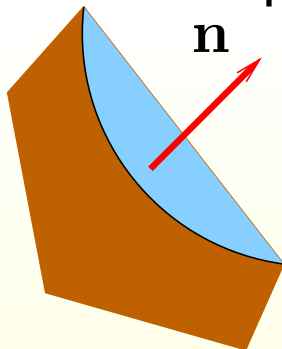
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contraintes initiales dans le solide : $\boldsymbol{\sigma}_{in}^s \neq 0$

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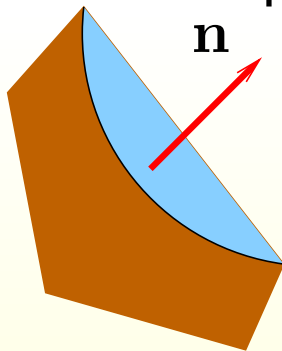
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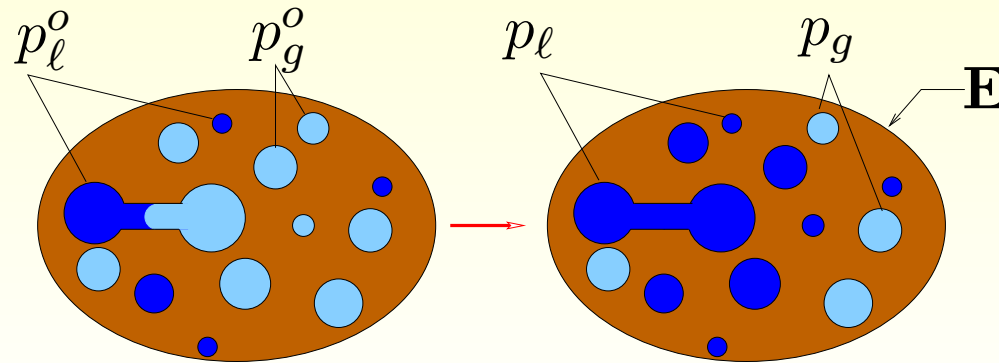
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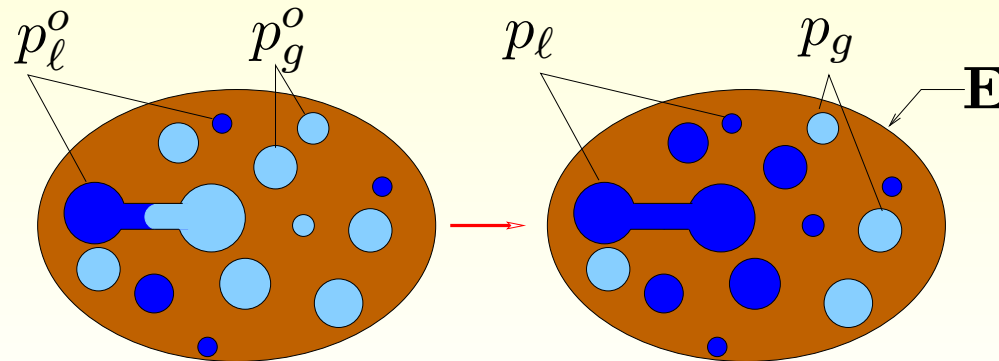
$$\boldsymbol{\Sigma}_{in} = \overline{\boldsymbol{\sigma}_{in}} = -p_g^o \mathbf{1}$$

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Equation d'état macroscopique



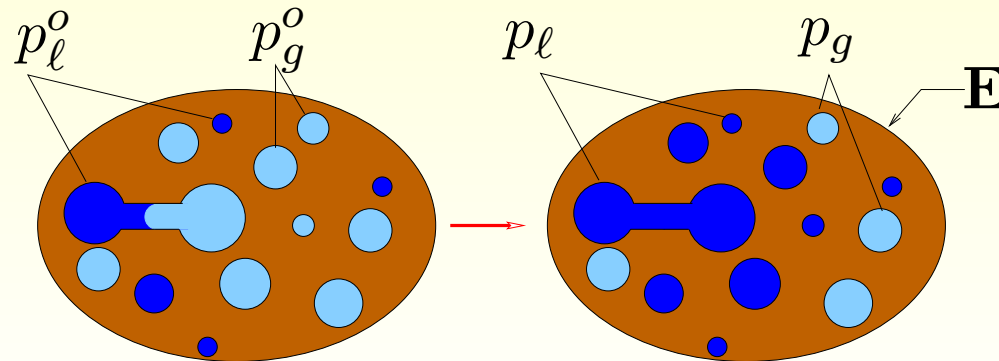
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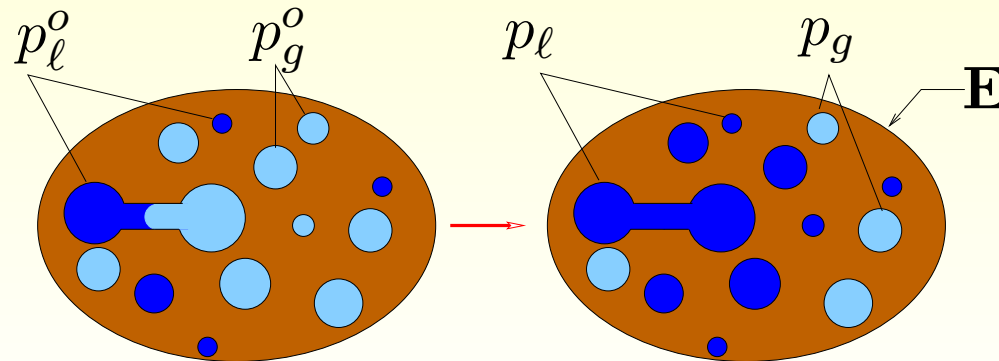
$$\boldsymbol{\sigma}_o(\mathbf{z}) = \begin{cases} \boldsymbol{\sigma}_{in}^s(\mathbf{z}) & (\Omega^s) \\ -p_l \mathbf{1} & (\Omega^\ell) \\ -p_g \mathbf{1} & (\Omega^g) \\ \gamma^{\alpha\beta} \mathbf{1}_T \delta_{\alpha\beta} & (I^{\alpha\beta}) \end{cases}$$

Equation d'état macroscopique



- en utilisant $\overline{\sigma_{in} : \mathbb{A}} = -p_g^o \mathbf{1}$ dans $\Sigma = \mathbb{C}^{hom} : \mathbf{E} + \overline{\sigma_o : \mathbb{A}} \dots$
- en introduisant $\bar{p} = \frac{\varphi^l}{\varphi} p^l + \frac{\varphi^g}{\varphi} p^g = S_r p^l + (1 - S_r) p^g$

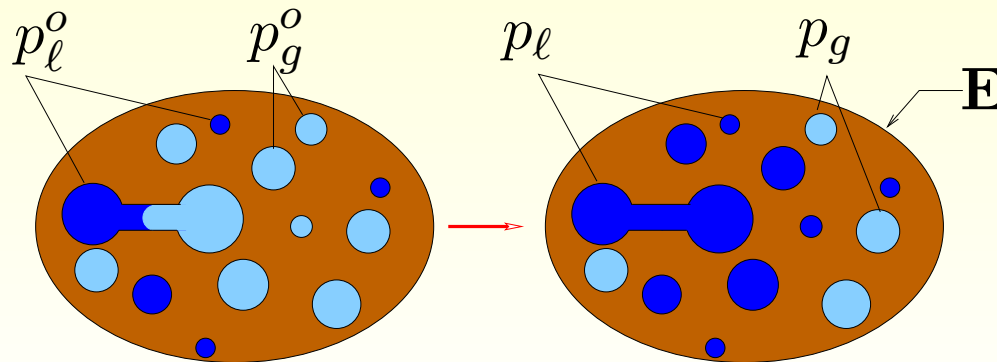
Equation d'état macroscopique



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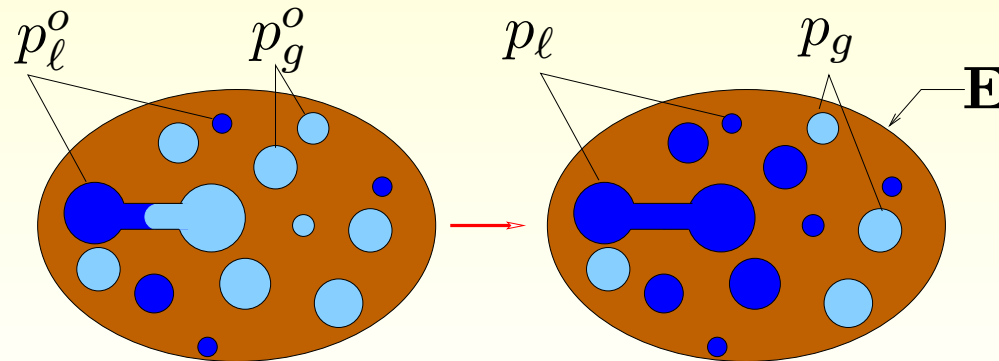
$$\delta \Sigma = \mathbb{C}^{hom} : \mathbf{E} + \delta \left(-\bar{p} \mathbf{B} + \left(\gamma^{\alpha\beta} \int_{I^{\alpha\beta}} \mathbf{1}_T : \mathbb{A} \frac{dS}{\mathcal{V}} \right) \right)$$

Equation d'état macroscopique



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Equation d'état macroscopique

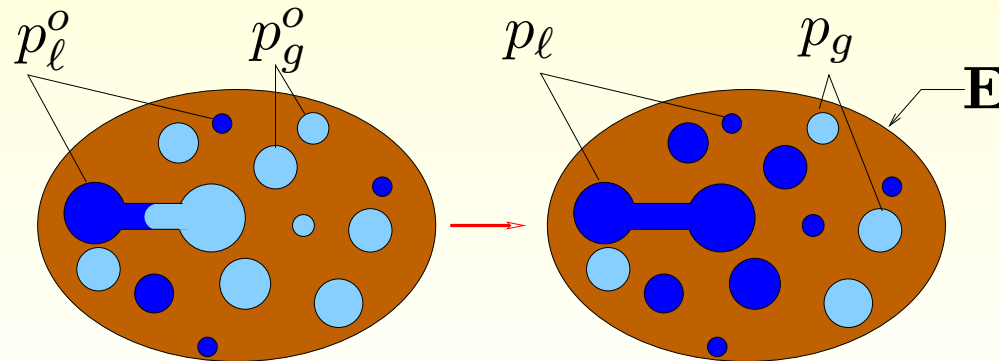


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gonflement/retrait libres :

$$\mathbf{E} = \mathbb{S}^{hom} : \delta \left(\bar{p} \mathbf{B} - \left(\gamma^{\alpha\beta} \int_{I^{\alpha\beta}} \mathbf{1}_T : \mathbb{A} \frac{dS}{\mathcal{V}} \right) \right)$$

Equation d'état macroscopique



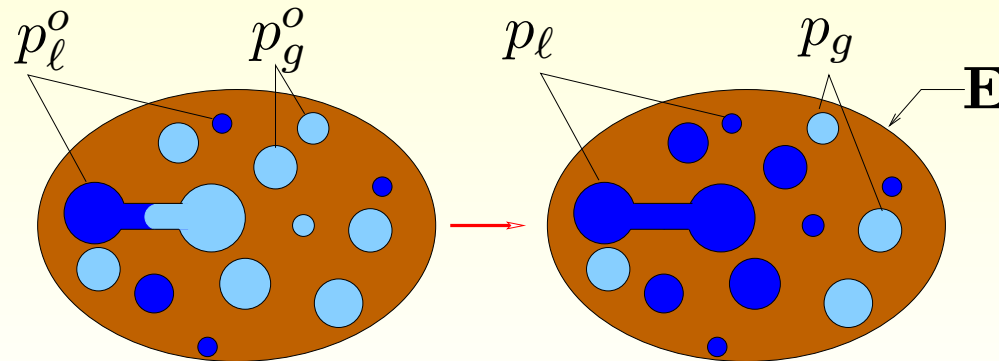
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“paramètres de chargement” : \bar{p} et non p_g ou p_l

Equation d'état macroscopique



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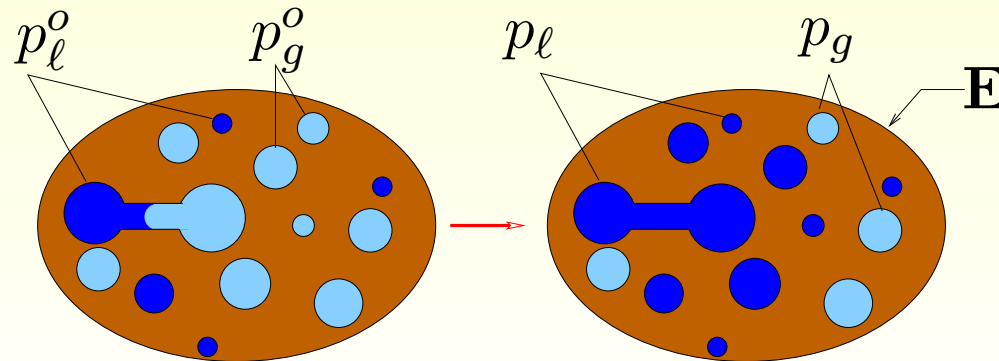
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Equation d'état macroscopique



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gonflement/retrait empêchés :

$$\delta \Sigma = \delta \left(-\bar{p} \mathbf{B} + \left(\gamma^{\alpha\beta} \int_{I^{\alpha\beta}} \mathbf{1}_T : \mathbb{A} \frac{dS}{\mathcal{V}} \right) \right)$$

“paramètres de chargement” : \bar{p} et non p_g ou p_l

$I^{\alpha\beta}$ et non S_r

Distribution isotrope de pores sphériques

- fraction volumique des pores de rayon $\in [r, r + dr]$ dans Ω^p :

$$\alpha(r) dr$$

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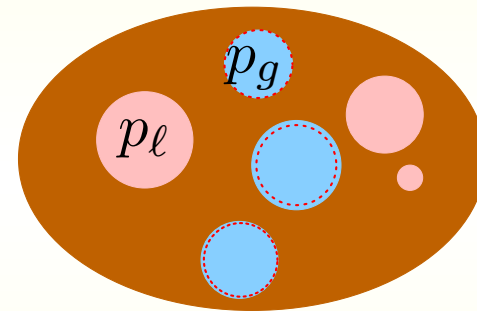
$$\int_0^{\infty} \alpha(r) dr = 1$$

Distribution isotrope de pores sphériques

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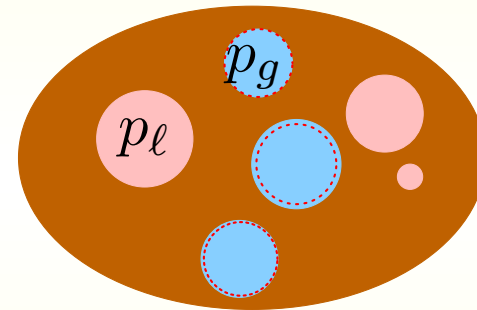
$$\int_0^{\infty} \alpha(r) dr = 1$$

- pores saturés en liquide ou en gaz

$r < r^*(S_r)$ pore saturé en liquide

$r > r^*(S_r)$ pore saturé en gaz

$$I^{sg} = \cup_{r > r^*(S_r)} \mathcal{S}(r)$$



Vers un concept de contrainte effective

$$-\bar{p}\mathbf{B} + \gamma^{\alpha\beta} \int_{I^{\alpha\beta}} \mathbf{1}_T : \mathbb{A} \frac{dS}{\mathcal{V}} = ?$$

Vers un concept de contrainte effective

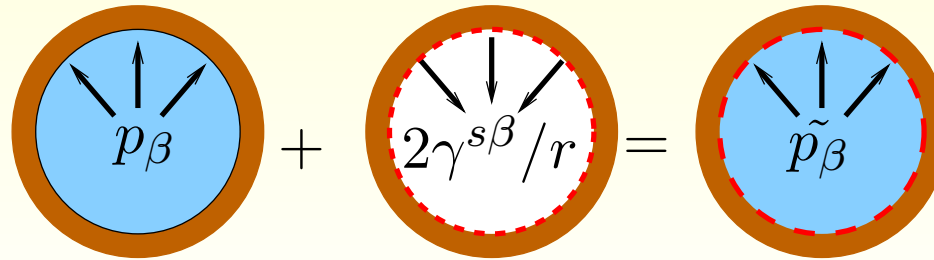
$$-\bar{p}\mathbf{B} + \gamma^{\alpha\beta} \int_{I^{\alpha\beta}} \mathbf{1}_T : \mathbb{A} \frac{dS}{\mathcal{V}} = -\mathbf{B} \int_0^\infty \tilde{p}_\beta(r) \alpha(r) dr$$

Vers un concept de contrainte effective

pression locale apparente : $\tilde{p}_\beta = p_\beta - \frac{2\gamma^{s\beta}}{r}$

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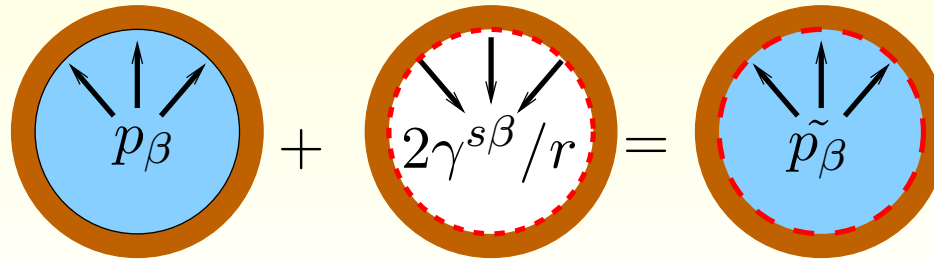
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Vers un concept de contrainte effective

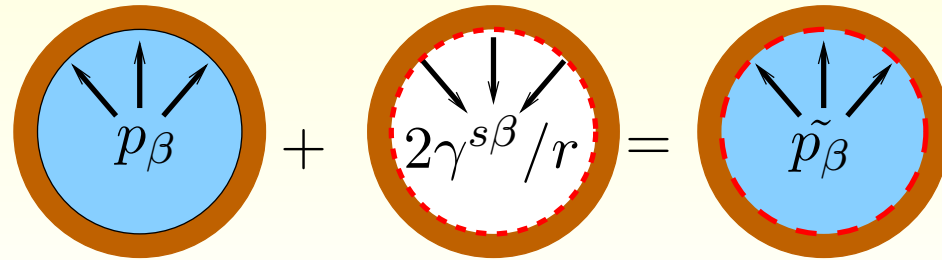


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Vers un concept de contrainte effective

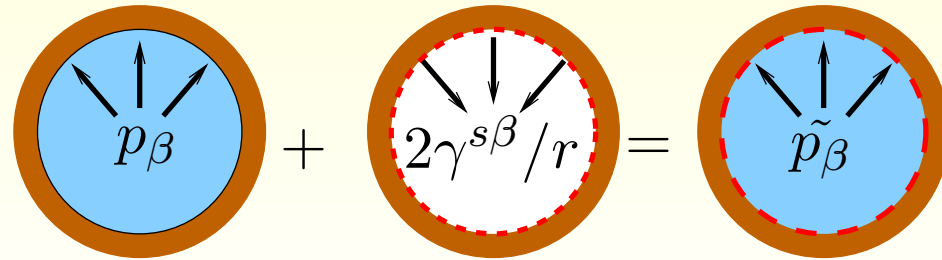


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$$\underbrace{\delta \left(-\bar{p}\mathbf{B} + \gamma^{\alpha\beta} \int_{I^{\alpha\beta}} \mathbf{1}_T : \mathbb{A} \frac{dS}{\nu} \right)}_{\delta\Sigma - \mathcal{C}^{hom} : \mathbf{E}} = -\mathbf{B} \delta \left(\underbrace{\int_0^\infty \tilde{p}_\beta \alpha(r) dr}_{P^{eq}(\mathbf{S}_r)} \right)$$

Vers un concept de contrainte effective



pression locale apparente : $\tilde{p}_\beta = p_\beta - \frac{2\gamma^{s\beta}}{r}$

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$$\delta \Sigma + \mathbf{B} \delta P^{eq} = \mathbb{C}^{hom} : \mathbf{E}$$

contrainte effective : $\delta \Sigma + \mathbf{B} \delta P^{eq}$

Application à l'essai de gonflement empêché

$$\delta\Sigma = -B\delta P^{eq}\mathbf{1}$$

Application à l'essai de gonflement empêché

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éprouvette de **bentonite** initialement non saturée

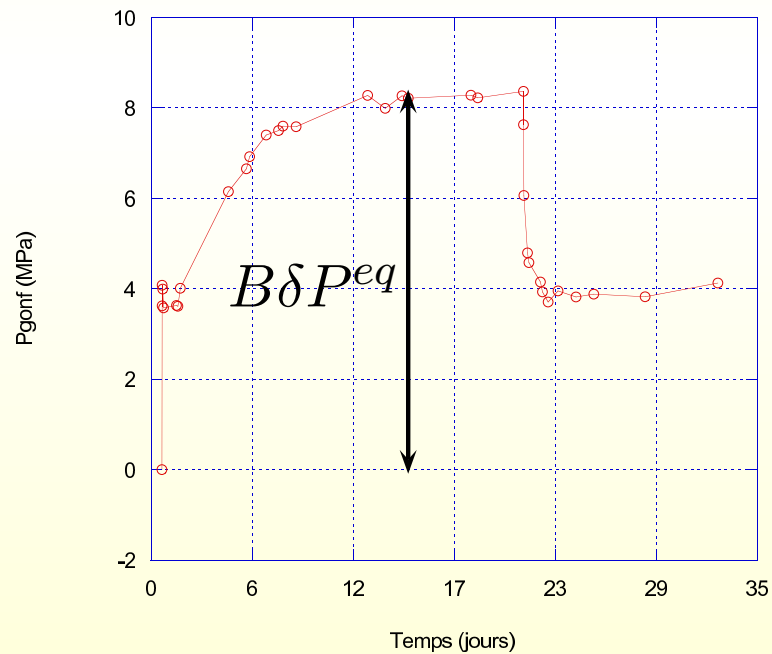
passage de l'état S_r^o à l'état saturé $S_r = 1$:

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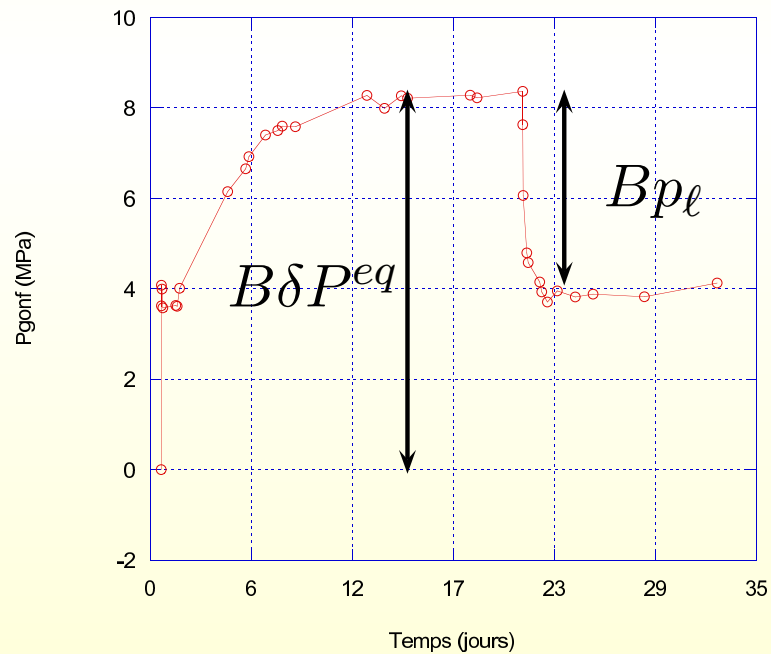


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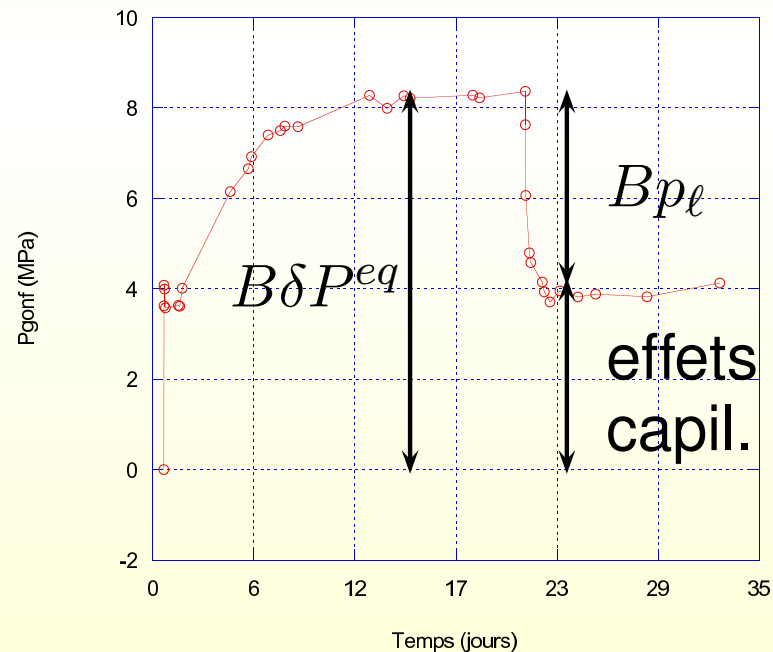


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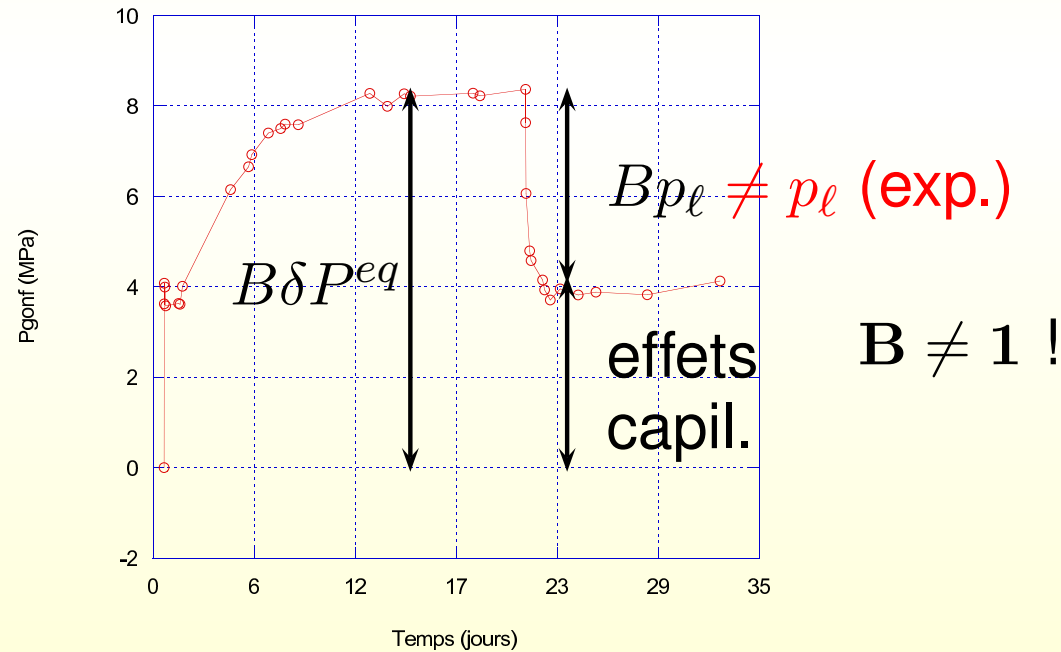


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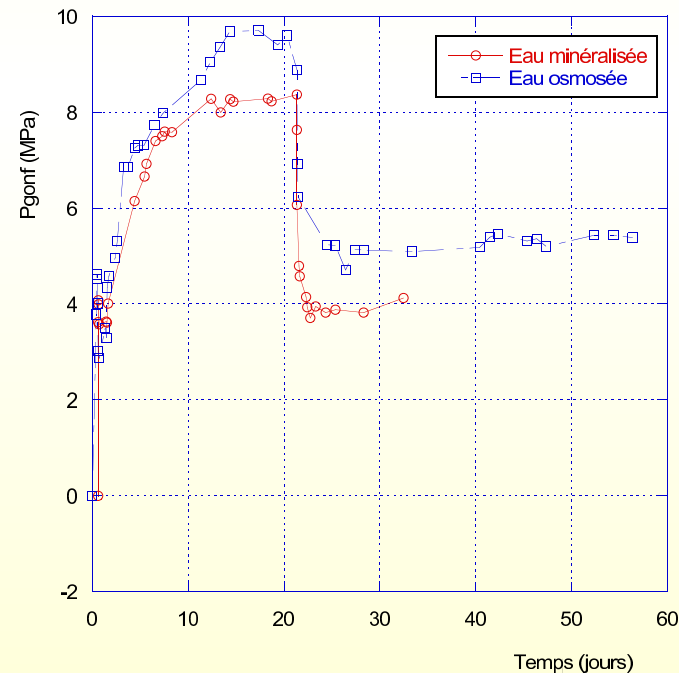


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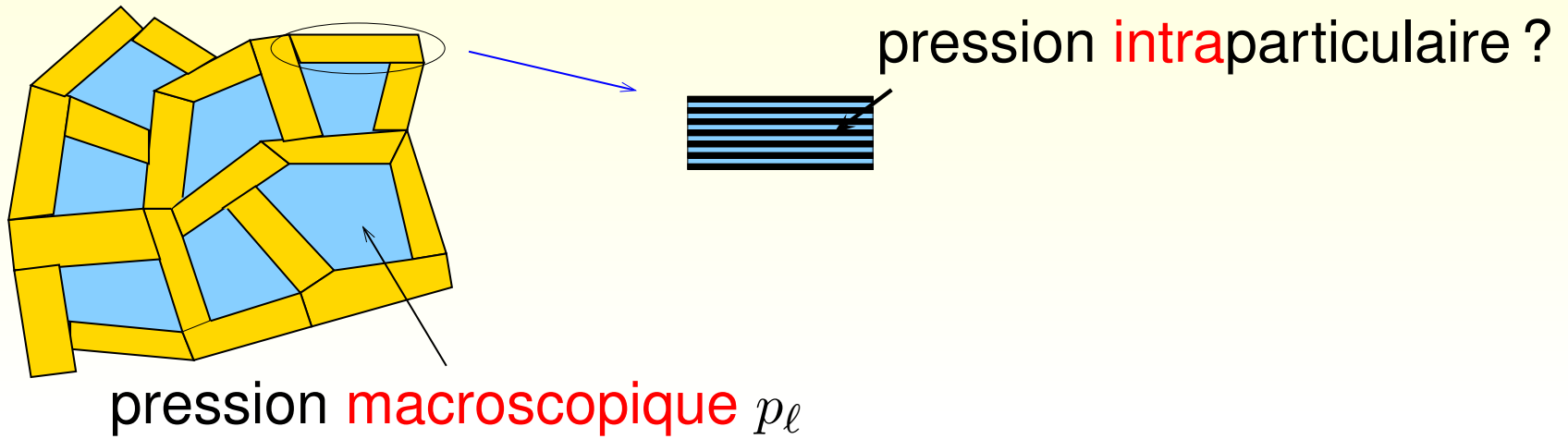
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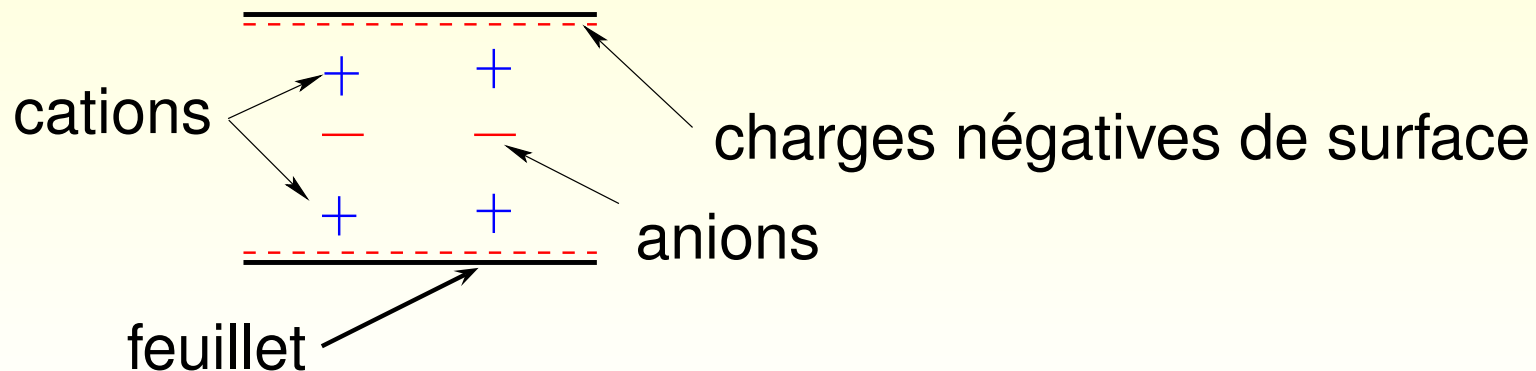


Un modèle à double porosité

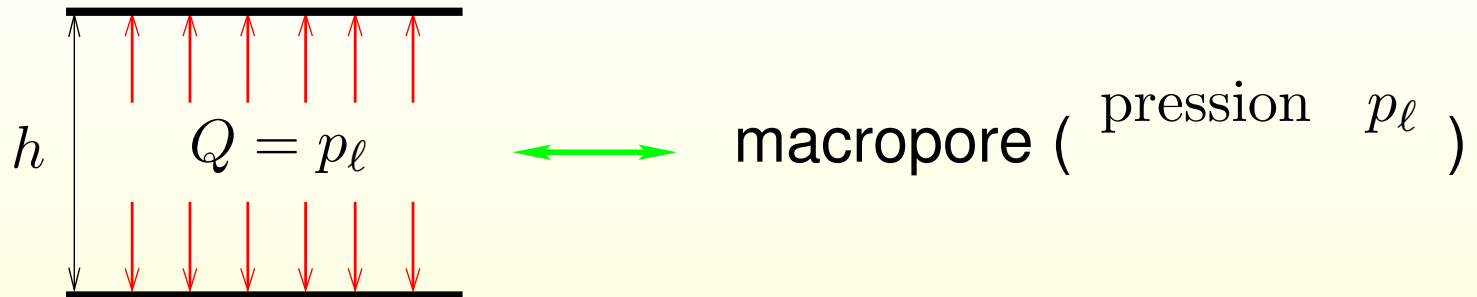
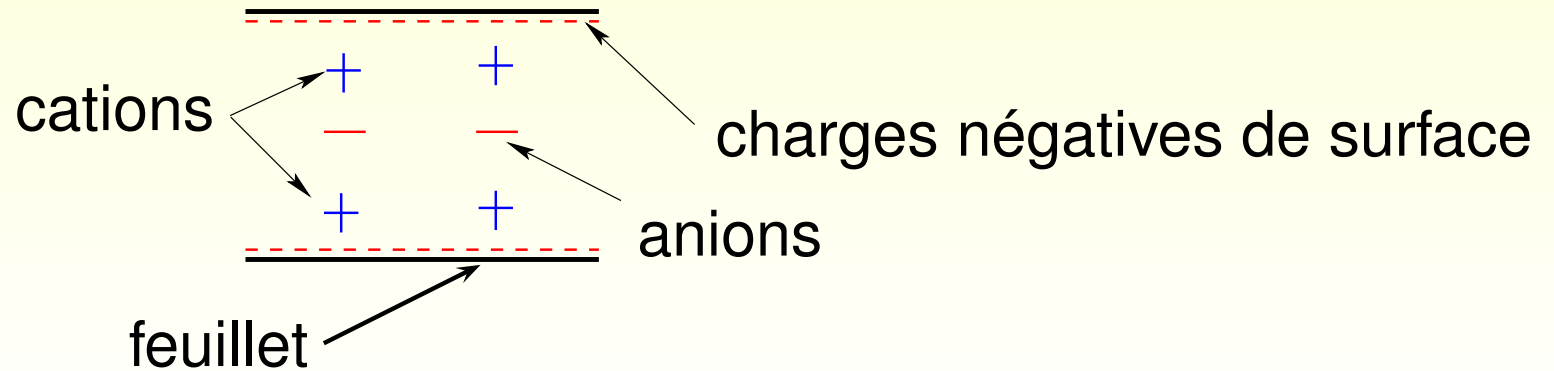
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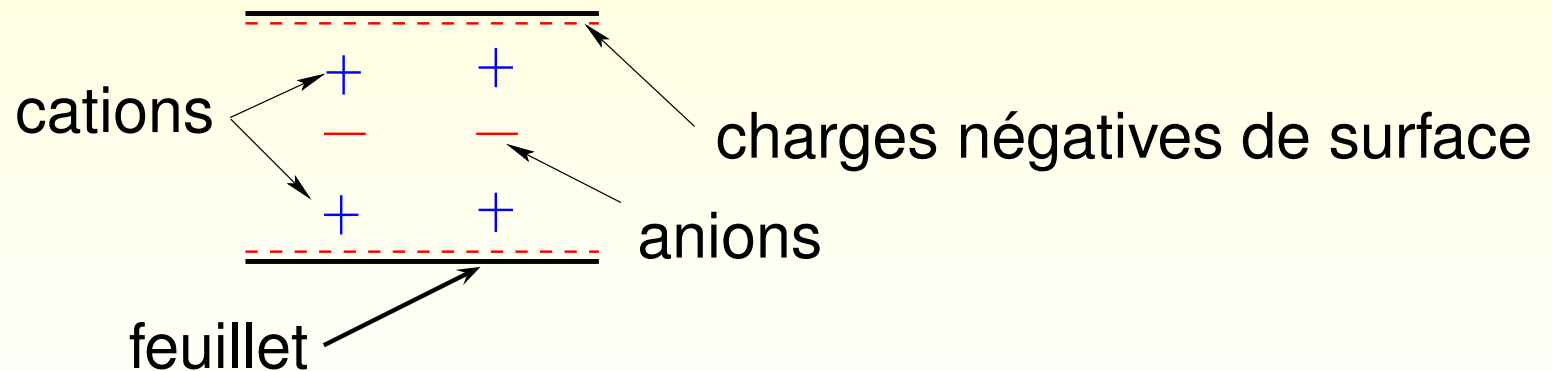
Interactions répulsives entre feuillets argileux



Interactions répulsives entre feuillets argileux

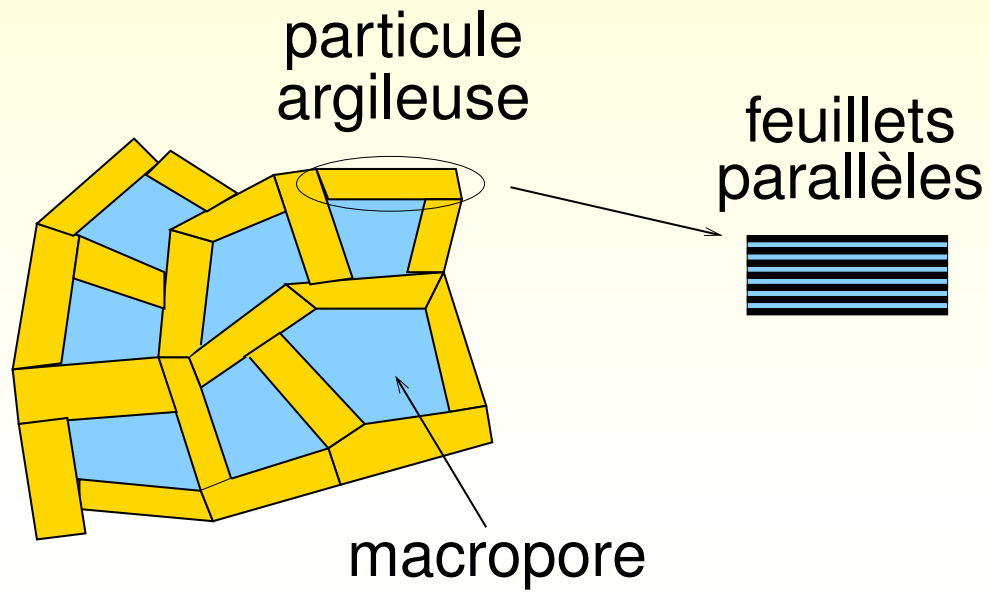


Interactions répulsives entre feuillets argileux

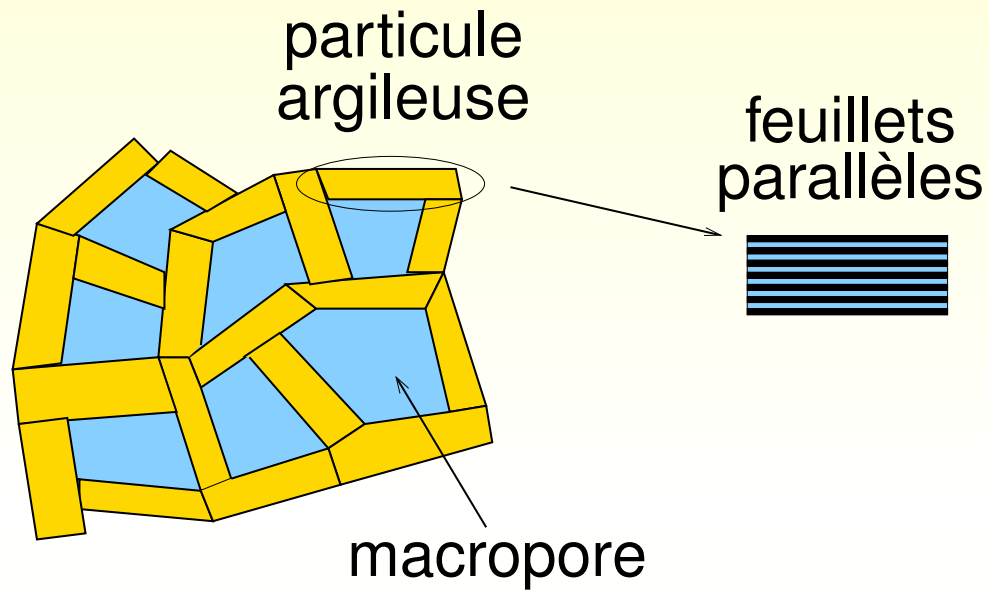


$$\pi^g = \mathcal{P}(h, n)$$

Echelles micro., méso. et macroscopiques



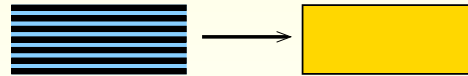
Echelles micro., méso. et macroscopiques



⇒ 2
niveaux
d'homogénéisation

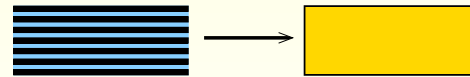
Echelles micro., méso. et macroscopiques

- 1er niveau (micro.→méso.)

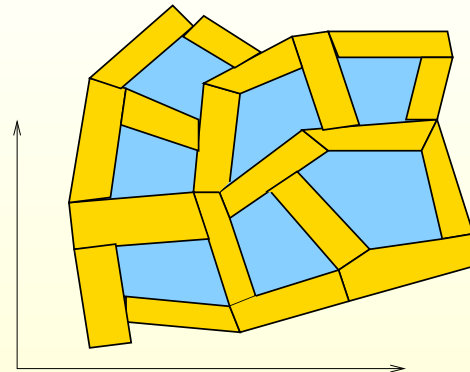


Echelles micro., méso. et macroscopiques

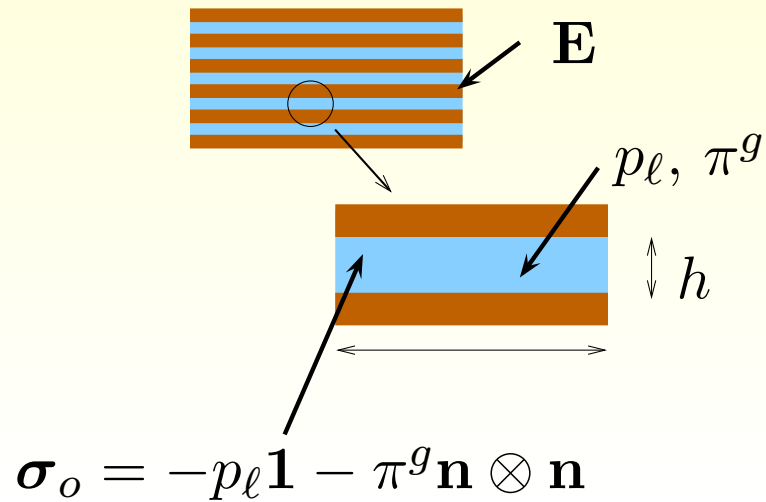
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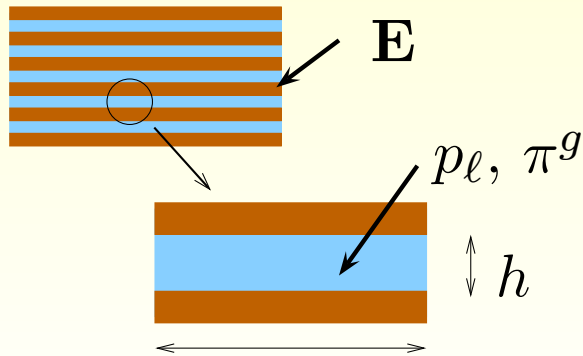
- 2ième niveau (méso.→macro.)



Niveau 1 : homogénéisation de la particule

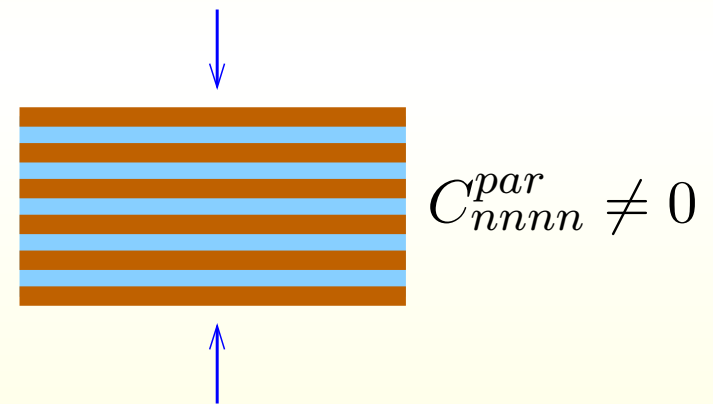
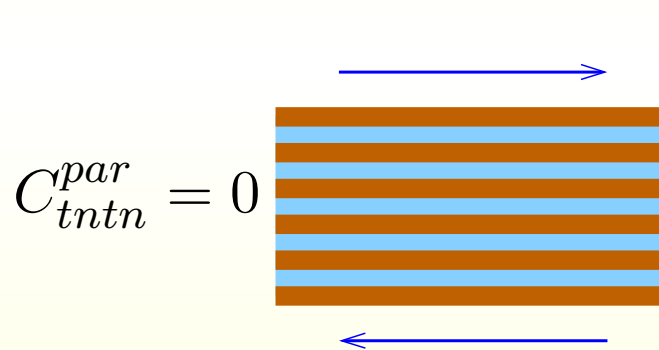


Niveau 1 : homogénéisation de la particule



$$\delta_n \pi^g(h_{in}) = \pi^g(h_{in}, n) - \pi_o^g(h_{in}, n_{in})$$

$$\delta \Sigma = \mathbb{C}^{par} : \mathbf{E} - \delta p_l \mathbf{1} - \delta_n \pi^g(h_{in}) \mathbf{B}_\pi$$

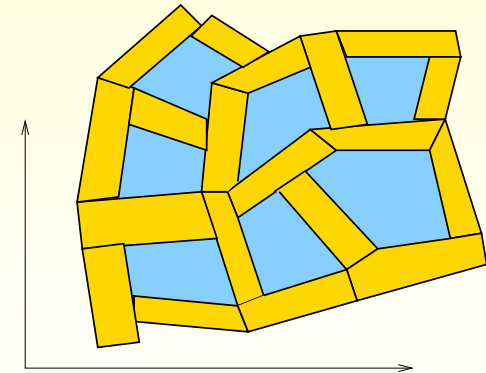


$$\mathbb{C}^{par} = \underbrace{\overline{\mathbb{C} : \mathbb{A}}}_{\text{meca.}} + \underbrace{\left(-\frac{h_{in}}{f} \frac{\partial \pi^g}{\partial h} \right)}_{\text{elec.}} \mathbf{B}_\pi \otimes \mathbf{B}_\pi$$

$$C_{nnnn}^{par} = -\frac{h_{in}}{f} \frac{\partial \pi^g}{\partial h} = \Pi > 0$$

Niveau 2 : distrib. isotrope particules/pores

$$\boldsymbol{\sigma} = \mathbb{C}(\mathbf{x}) : \boldsymbol{\varepsilon} + \boldsymbol{\sigma}_o(\mathbf{x})$$

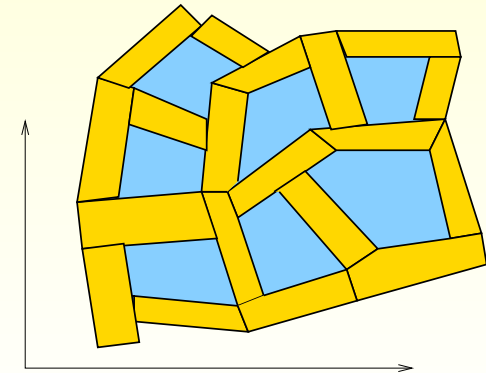


$$\left\{ \begin{array}{l} \boldsymbol{\sigma} = \mathbb{C}^{par} : \boldsymbol{\varepsilon} + \boldsymbol{\sigma}_{in} - \delta p_\ell \mathbf{1} - \delta_n \pi^g \mathbf{B}_\pi \quad (\text{particules argileuses}) \\ \boldsymbol{\sigma} = -p_\ell \mathbf{1} \quad (\text{macropores}) \end{array} \right.$$

Niveau 2 : distrib. isotrope particules/pores

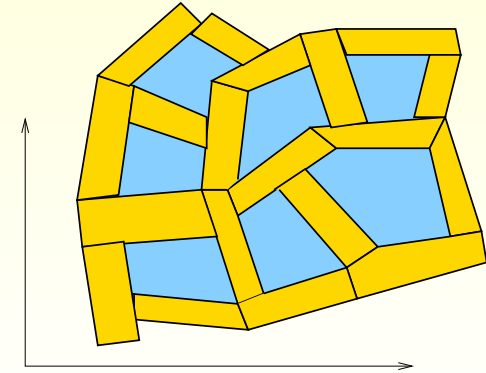
$$\boldsymbol{\sigma} = \mathbb{C}(\mathbf{x}) : \boldsymbol{\varepsilon} + \boldsymbol{\sigma}_o(\mathbf{x})$$

$$\mathbb{C}(\mathbf{x}) = \begin{cases} \mathbb{C}^{par}(\mathbf{x}) \\ 0 \end{cases} \quad \boldsymbol{\sigma}_o(\mathbf{x}) = \begin{cases} \boldsymbol{\sigma}_{in} - \delta p_\ell \mathbf{1} - \delta_n \pi^g \mathbf{B}_\pi(\mathbf{x}) & \text{(particules)} \\ -p_\ell \mathbf{1} & \text{(macropores)} \end{cases}$$



Niveau 2 : distrib. isotrope particules/pores

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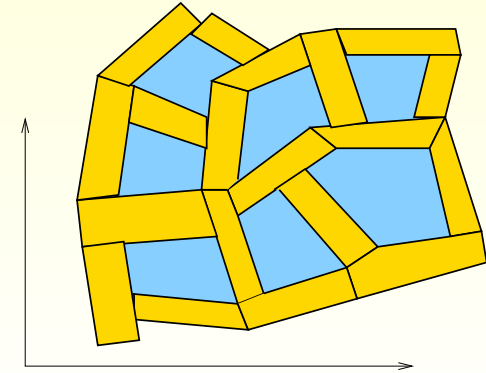


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$$\boldsymbol{\Sigma} = \overline{\mathbb{C}} : \overline{\mathbb{A}} : \mathbf{E} + \overline{\boldsymbol{\sigma}_o} : \overline{\mathbb{A}}$$

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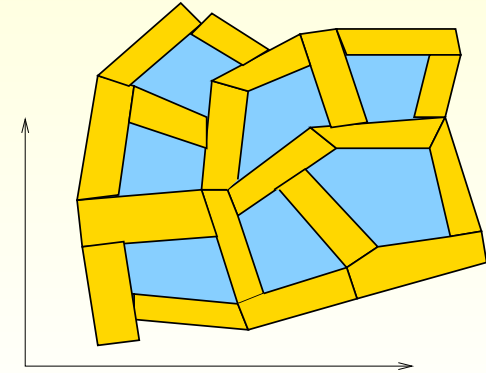


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$$\delta \boldsymbol{\Sigma} + (\delta p_\ell + g(\phi) \delta_n \pi^g) \mathbf{1} = \mathbb{C}^{ac}(\phi, \Pi) : \mathbf{E}$$

Niveau 2 : distrib. isotrope particules/pores

$$\boldsymbol{\sigma} = \mathbb{C}(\mathbf{x}) : \boldsymbol{\varepsilon} + \boldsymbol{\sigma}_o(\mathbf{x})$$

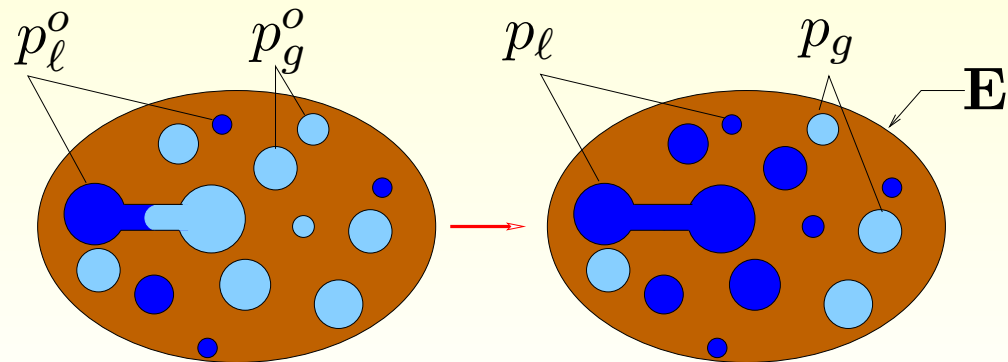


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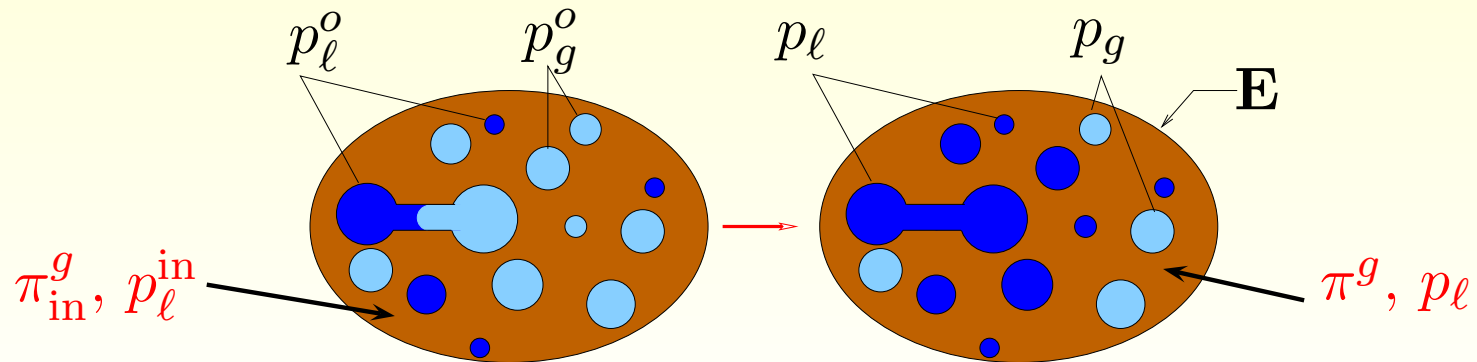
$$\delta \Sigma + (\delta p_\ell + g(\phi) \delta_n \pi^g) \mathbf{1} = \mathbb{C}^{ac}(\phi, \Pi) : \mathbf{E}$$

pression de gonflement : $\delta p_\ell + g(\phi) \delta_n \pi^g$

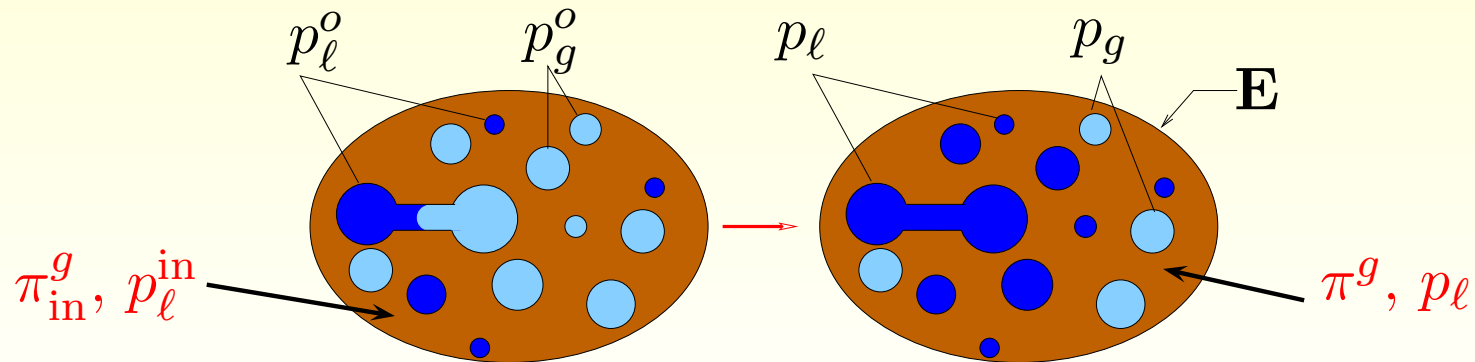
Généralisation au cas non saturé



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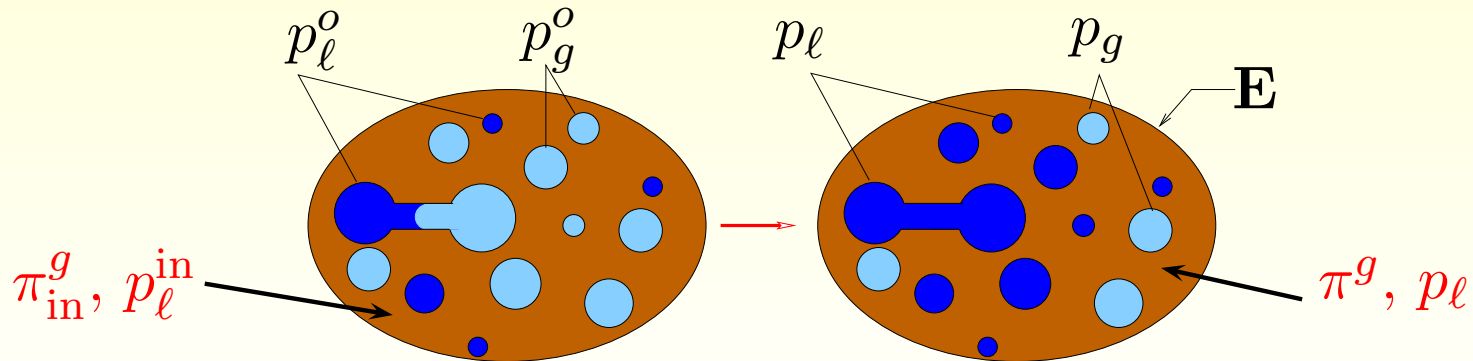


- effet de la resaturation sur la salinité

$$n \times S_r = \text{Const.}$$

$$S_r \uparrow \Rightarrow n \downarrow \Rightarrow \pi^g \uparrow$$

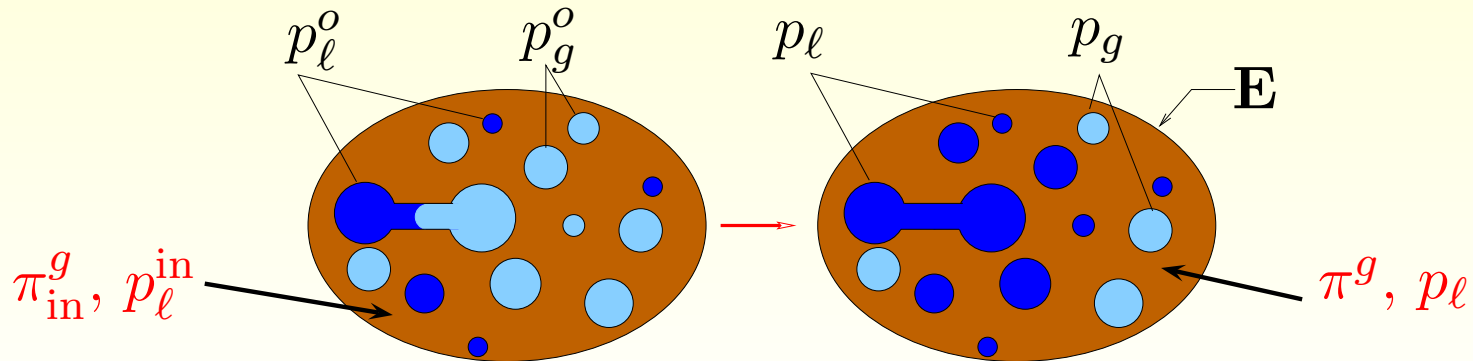
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$$\boldsymbol{\sigma} = \mathbb{C}(\mathbf{x}) : \boldsymbol{\varepsilon} + \boldsymbol{\sigma}_o(\mathbf{x})$$

$$\mathbb{C}(\mathbf{x}) = \begin{cases} \mathbb{C}^{par}(\mathbf{x}) \\ 0 \\ 0 \\ 0 \end{cases} \quad \boldsymbol{\sigma}_o(\mathbf{x}) = \begin{cases} \boldsymbol{\sigma}_{in} - \delta p^l \mathbf{1} - \delta_n \pi^g \mathbf{B}_\pi(\mathbf{x}) & (\text{particules}) \\ -p^l \mathbf{1} & (\Omega^l) \\ -p^g \mathbf{1} & (\Omega^g) \\ \gamma^{\alpha\beta} \mathbf{1}_T \delta_{I\alpha\beta} & I\alpha\beta \end{cases}$$

Généralisation au cas non saturé

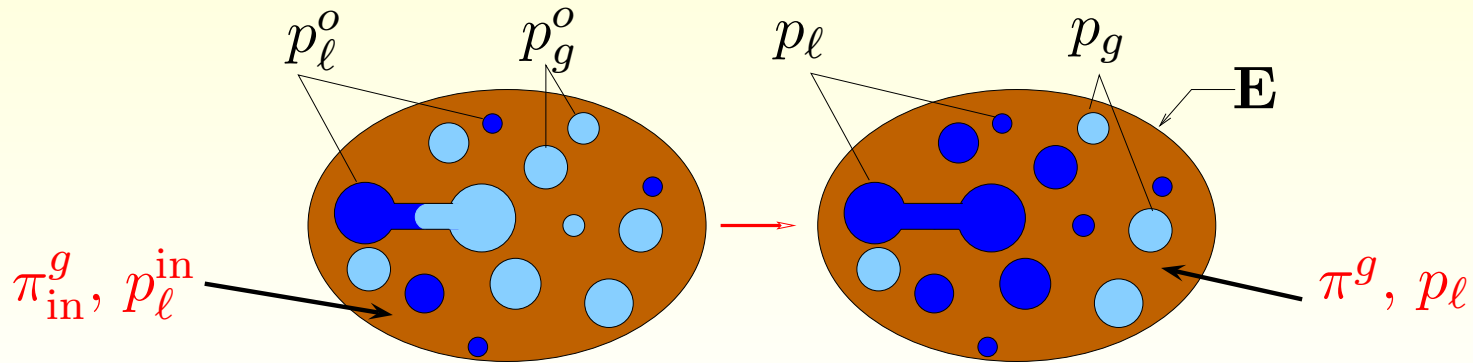


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$$\boldsymbol{\Sigma} = \overline{\mathbb{C}} : \overline{\mathbb{A}} : \mathbf{E} + \overline{\boldsymbol{\sigma}_o} : \overline{\mathbb{A}}$$

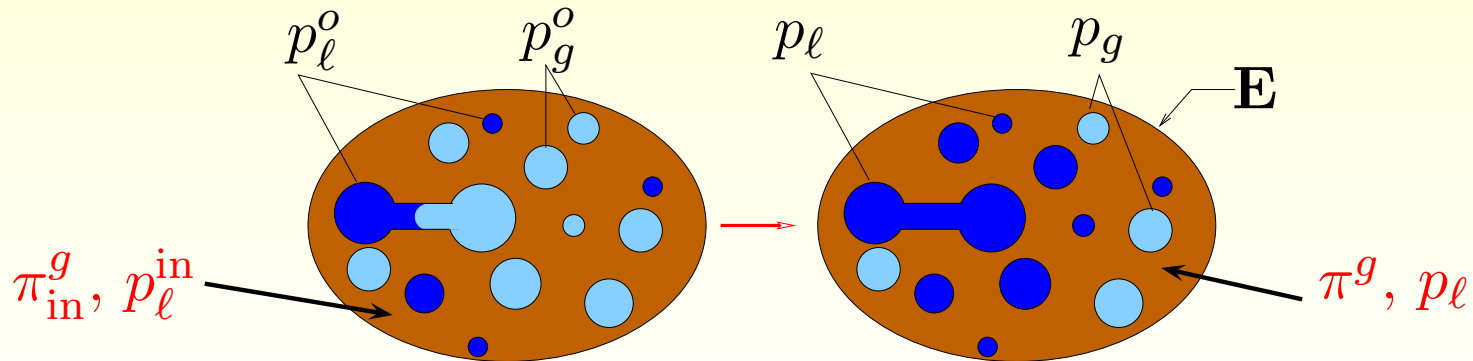
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$$\delta \boldsymbol{\Sigma} = \mathbb{C}^{ac} : \mathbf{E} + \delta \left(-\bar{p} \mathbf{B} + \left(\gamma^{\alpha\beta} \int_{I^{\alpha\beta}} \mathbf{1}_T : \mathbb{A} \frac{dS}{\mathcal{V}} \right) \right) - g(\phi) \delta_n \pi^g \mathbf{1} - \delta p^\ell (\mathbf{1} - \mathbf{B})$$

Généralisation au cas non saturé



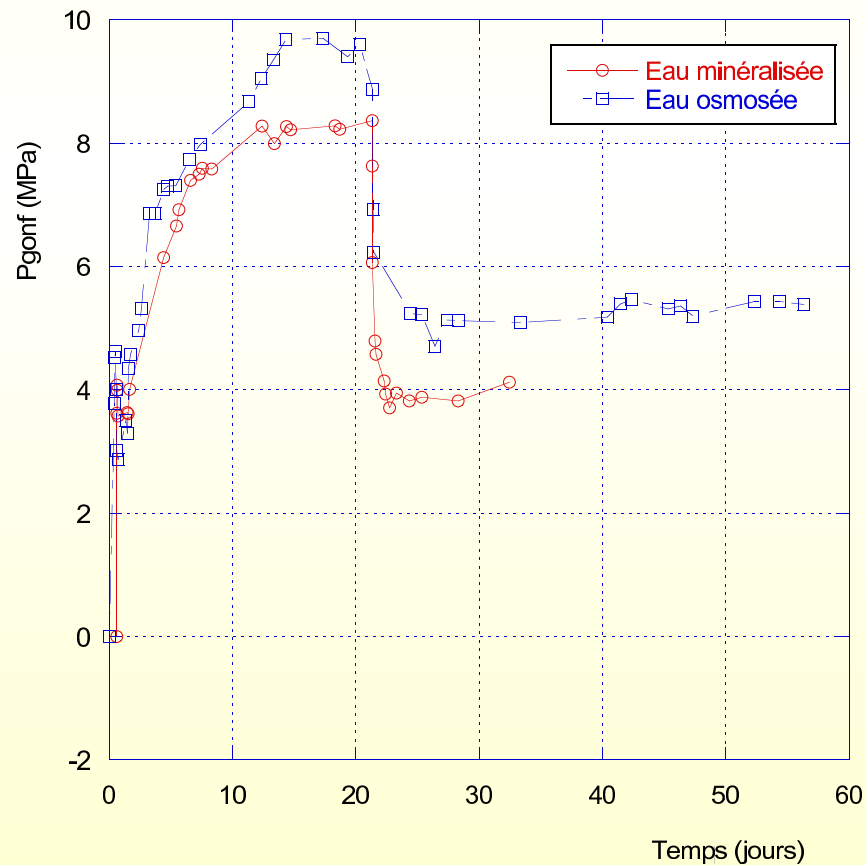
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- pores sphériques :

$$\delta \boldsymbol{\Sigma} = \mathbb{C}^{ac} : \mathbf{E} - B \delta P^{eq} \mathbf{1} - g(\phi) \delta_n \pi^g \mathbf{1} - \delta p^\ell (1 - B) \mathbf{1}$$

Conclusion : retour à l'expérience

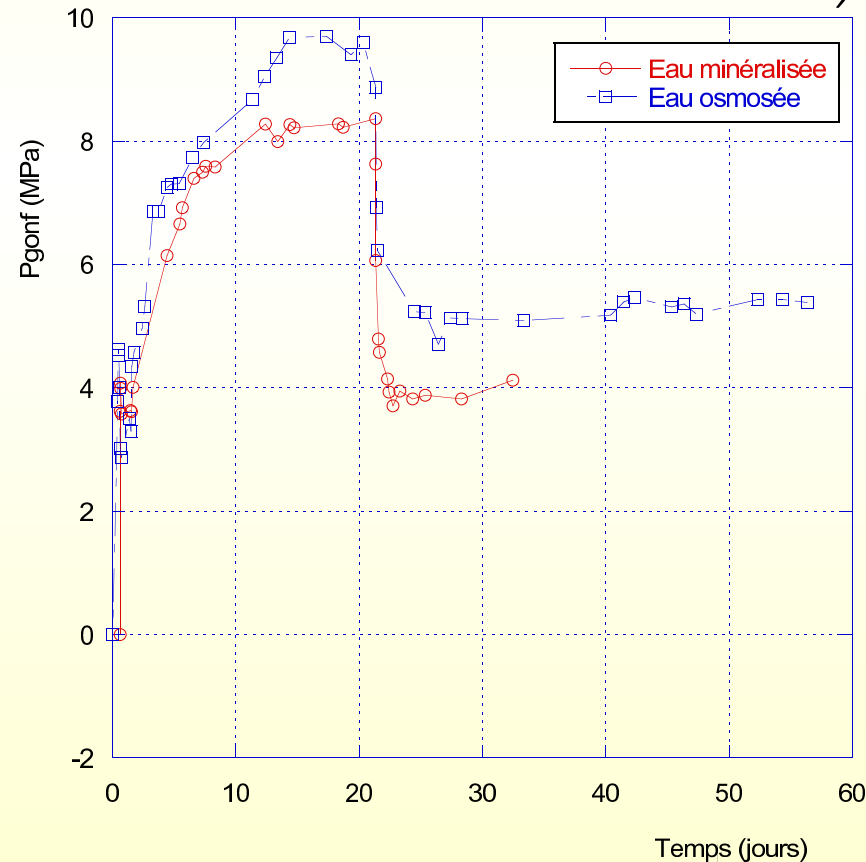
$$\delta\Sigma = -B\delta P^{eq}\mathbf{1} - \delta p^{\ell}(1 - B)\mathbf{1} - g(\phi)\delta_n\pi^g\mathbf{1}$$



Conclusion : retour à l'expérience

- pression de gonflement : (de l'état S_r^o à l'état saturé $S_r = 1$)

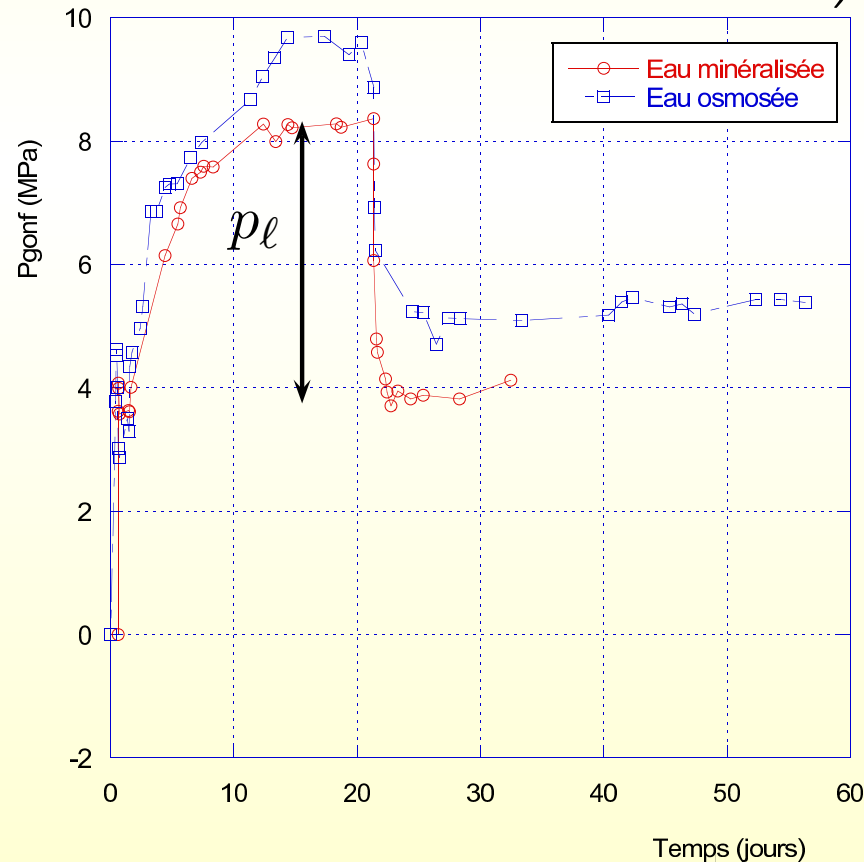
$$p_e - p_\ell^o + B \left(2\gamma^{lg} \int_{r^*(S_r^o)}^{\infty} \frac{\alpha(r)}{r} dr - p_g^o(1 - S_r^o) \right) + g(\phi)\delta_n\pi^g$$



Conclusion : retour à l'expérience

- pression de gonflement : (de l'état S_r^o à l'état saturé $S_r = 1$)

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