THERMOMECHANICS OF UNDERGROUND CAVITIES

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Abstract. The aim of this presentation is to underline the common features of the models builded for a circular tunnel, performed in an elastoplastic Tresca material, subjected to a thermal field, in the cases of constant, respectively decreasing with temperature cohesion. Our models are consistent, observing the same order of appearance of elastoplastic zones.

Keywords : *thermoplasticity*, *elastoplastic zones*, *underground cavities*

1 Introduction

In recent years the problem of thermomechanical evolution of underground structures have received considerable attention. Many practical problems can be treated by this type of modelling such as underground storage of nuclear wastes, extraction of geothermal energy, underground coal gazification, etc.

The case of a tunnel in an elastic-perfectly-plastic medium, subjected to a thermo-mechanical loading was treated in [1], [2], and [3, 4], leading to explicit solutions. In [5] the geometry of the problem was changed, obtaining an analytical model of an elastoplastic thick-walled tube subjected to an internal pressure and to an axisymmetrical time-dependent temperature field.

Furthermore, there are clear evidences that the resistance of geomaterials decreases when the temperature rises, which have been confirmed by experiments in different geological formations. These experimental observations have justified the development of a thermo-mechanical model capable of simulating such thermal- softening behaviour, as in [6]. Despite the dependence of cohesion on temperature, we found a closed form solution. A synthesis of the previous cited works can be found in [7].

The aim of the present paper is to underline the common features of the models builded for a circular tunnel, performed in an elastoplastic material obeying Tresca's yield criterion and its associated flow rule, subjected to an axisymmetrical unsteady thermal field, in the cases of constant, respectively

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decreasing with temperature cohesion. Our models are consistent, observing the same order of appearance of elastoplastic zones.

2 Fundamental equations

We consider an infinite, isotropic, homogeneous, elasto-perfectly plastic medium, obeying Tresca's yield criterion and its associated flow rule. At the time $t = 0$, the medium is supposed to be in a state of hydrostatic stress $\boldsymbol{\sigma_0} = -P_\infty \mathbf{1}$ with zero displacement $\boldsymbol{u_0} = 0$ and zero strain $\boldsymbol{\varepsilon_0} = 0$ everywhere, and having a reference temperature T_0. We suppose that all the material properties are temperature independent, excepting the cohesion.

The cohesion at the reference temperature is denoted by $C_0 = C(T_0)$. In order to take into account the experimental evidences, it is proposed to introduce the dependence of the cohesion on temperature as follows:

$$C = C(T), \text{ with } \frac{\partial C}{\partial T} < 0. \tag{1}$$

We are considering a quasi-static evolution, under the hypothesis of small axisymmetrical plane strains. Under the previous assumptions the displacement is purely radial:

$$\boldsymbol{u} = u(r,t)\boldsymbol{e}_r \tag{2}$$

and consequently, the stress and strain tensor are diagonal:

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \partial_r u(r,t) & & \\ & u(r,t)/r & \\ & & 0 \end{pmatrix}, \ \boldsymbol{\sigma} = \begin{pmatrix} \sigma_r(r,t) & & \\ & \sigma_\theta(r,t) & \\ & & \sigma_z(r,t) \end{pmatrix}. \tag{3}$$

We are imposing in the wall an internal pressure, which decreases monotonously from its initial value P_∞ to a final constant prescribed value P_i. We shall assume that the medium remains elastic during this stage of mechanical loading.

Maintaining P_i at its previous prescribed value, the tunnel is subjected to a heating process, resulting in an axisymmetrical temperature field $T(r,t)$. We shall restrict our attention to the case of a temperature field satisfying the following essential conditions:

$$\partial_r T(r,t) < 0 \ , \ \partial_t T(r,t) > 0. \tag{4}$$

Such is the case, for example, of a constant heat flux applied at the tunnel wall [8].

On the other hand, as the temperature is a function of radius r and time t, the cohesion $C(T) = C(t, r)$ depends on same variables. Moreover, from the inequalities (1) and (4) we obtain :

$$\partial_r C(r,t) > 0,\ \partial_t C(r,t) < 0. \tag{5}$$

Under the previous assumptions, the constitutive equation takes the form:

$$\partial_t \boldsymbol{\varepsilon} \ = \frac{1+\nu}{E}\, \partial_t \boldsymbol{\sigma} \ - \frac{\nu}{E}\, tr(\partial_t \boldsymbol{\sigma}) + \alpha\, \partial_t T\, \mathbf{1} + \partial_t \boldsymbol{\varepsilon}^p \tag{6}$$

where ν is Poisson's ratio, E the Young's modulus, α the coefficient of linear thermal expansion and $T(r,t)$ the temperature field.

The form of the plastic strain rate tensor $\partial_t \boldsymbol{\varepsilon}^p$ depends on whether we are in the presence of face flow or corner flow:

$$\partial_t \boldsymbol{\varepsilon}^p = \left[\begin{array}{ll} \partial_t \lambda \, \dfrac{\partial F}{\partial \boldsymbol{\sigma}} & \text{if} \quad \sigma_i > \sigma_j > \sigma_k \\ & \\ \partial_t \lambda_{ij} \, \dfrac{\partial F_{ij}}{\partial \boldsymbol{\sigma}} + \partial_t \lambda_{ik} \, \dfrac{\partial F_{ik}}{\partial \boldsymbol{\sigma}} & \text{if} \quad \sigma_i > \sigma_j = \sigma_k \end{array} \right. \tag{7}$$

where $F = \sigma_i - \sigma_k - 2C(r,t)$ is the Tresca's yield criterion (in the case of face flow), respectively $F_{lm} = |\sigma_l - \sigma_m| - 2C(r,t)$ (in the case of corner flow). Here:

$$\partial_t \lambda = \left[\begin{array}{l} 0 \quad \text{if} \quad F < 0 \quad \text{or} \quad \partial_t \boldsymbol{\sigma} \cdot \dfrac{\partial F}{\partial \boldsymbol{\sigma}} < 0 \\ \\ > 0 \quad \text{if} \quad F = 0 \quad \text{and} \quad \partial_t \boldsymbol{\sigma} \cdot \dfrac{\partial F}{\partial \boldsymbol{\sigma}} = 0 \end{array} \right.$$

and the sign of $\partial_t \lambda_{lm}$ equals the sign of $\sigma_l - \sigma_m$.

The quantities $E, \sigma_r, \sigma_\theta, \sigma_z, P_\infty, P_i$ will be normalized with respect to the cohesion C_0 and we shall denote by:

$$\theta(r,t) = \frac{E\alpha T(r,t)}{2C_0(1-\nu)} \qquad \textit{–dimensionless thermal loading}$$

$$\Delta P = \frac{P_\infty - P_i}{C_0} \qquad \textit{–dimensionless mechanical loading}$$

$$\mathcal{C}(r,t) = \frac{C(r,t)}{C_0} \qquad \textit{–dimensionless cohesion.}$$

Integrating the constitutive equation (6), with respect to time between $t = 0$ and any other instant $t > 0$ and taking into account the form (7) of $\partial_t \varepsilon^p$ we obtain the fundamental constitutive equations:

$$\begin{gathered} E\partial_r u = \sigma_r - \nu(\sigma_\theta + \sigma_z) + 2(1-\nu)\theta + E\lambda + E\mu + (1-2\nu)P_\infty \\ E\, u/r = \sigma_\theta - \nu(\sigma_z + \sigma_r) + 2(1-\nu)\theta - E\lambda + (1-2\nu)P_\infty \qquad (8) \\ 0 = \sigma_z - \nu(\sigma_r + \sigma_\theta) + 2(1-\nu)\theta - E\mu + (1-2\nu)P_\infty \end{gathered}$$

to which must be added the equilibrium equation:

$$\sigma_\theta - \sigma_r = r\, \partial_r \sigma_r \qquad (9)$$

and the Tresca's yield condition:

$$PF\ (face\ flow) \qquad \sigma_i - \sigma_k = 2\mathcal{C}(r,t) \quad (\text{if } \sigma_i > \sigma_j > \sigma_k) \qquad (10)$$

$$PC\ (corner\ flow) \qquad \sigma_i - \sigma_k = 2\mathcal{C}(r,t)\ ,\ \sigma_j = \sigma_k \quad (\text{if } \sigma_i > \sigma_j = \sigma_k). \qquad (11)$$

Note that in the system (8) the multipliers λ and μ are associated with the stress couples $\sigma_r - \sigma_\theta$, $\sigma_r - \sigma_z$ being non-zero only in the case of plastic flow.

The boundary conditions are:

$$\sigma_r(a,t) = -P_i \text{ and } \lim_{r\to\infty} \sigma_r(r,t) = -P_\infty. \qquad (12)$$

To solve the system (8)–(12), a sequence of elastoplastic zones is assumed, for each phase encountered. The solution so established is verified, a posteriori, for consistency with respect to the following conditions:

- the boundary radii must be monotone increasing with time;
- the signs of $\partial_t \lambda$ and $\partial_t \mu$ must be the same as the corresponding differences $\sigma_r - \sigma_\theta$, respectively $\sigma_r - \sigma_z$ in each plastic zone, so that the plastic power is positive;
- the deviatoric stresses must stay below the yield limit in the elastic zone.

3 Evolution of elastoplastic zones

3.1 Stage 1: Mechanical loading

In this phase the plastic multipliers and the temperature are taken as zero in Eqs. (8)-(10). The solution is found to be the classical elastic one. For the simplicity we suppose that in this stage the medium remains elastic, i.e. $\Delta P < 1$.

3.2 Stage 2: Thermal loading

The order of appearance of the elastoplastic zones is the same as in the case of constant cohesion. The moment $t = t_1$, when the yield limit is reached at the tunnel wall $r = a$, is defined by:

$$\theta(a, t_1) - \mathcal{C}(a, t_1) = -\Delta P. \tag{13}$$

Phase 2 ends at the moment $t = t_2$, when $\sigma_z(a, t_2) = \sigma_\theta(a, t_2)$. This occurs when:

$$\theta(a, t_2) - \mathcal{C}(a, t_2) = -\frac{1 - 2\nu}{2(1 - \nu)}\Delta P. \tag{14}$$

Finally, phase 3 ends at the moment $t = t_3$, when:

$$\theta(x(t_3), t_3) - \mathcal{C}(x(t_3), t_3) = 0 \tag{15}$$

where $x(t)$ is the elastoplastic radius.

In *Table* 1 we present the evolution of plastic zones during the thermal loading, this behaviour being present for the thermomechanical models of a circular tunnel obtained by us, i.e. for a Tresca yield criterion, and for a thermal softening material with Tresca yield criterion. The same evolution was found for a non-associated Coulomb elastoplastic model (see [2]).

Here the plastic zones PF_1, PF_2, and PC are characterized by the stress components order $\sigma_r > \sigma_z > \sigma_\theta$, $\sigma_r > \sigma_\theta > \sigma_z$, resp. $\sigma_r > \sigma_z = \sigma_\theta$. Comparisons with numerical values can be found in [4] and [7].

Phase 1: Elastic medium $EL(a, \infty)$	$0 < t < t_1$
Phase 2: Elastoplastic medium $PF_1(a, x)$, $EL(x, \infty)$	$t_1 < t < t_2$
Phase 3: Elastoplastic medium $PC(a, y)$, $PF_1(y, x)$, $EL(x, \infty)$	$t_2 < t < t_3$
Phase 4: Elastoplastic medium $PC(a, y)$, $PF_2(y, x)$, $EL(x, \infty)$	$t_3 < t$

Table 1: Evolution of plastic zones during thermal loading

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